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# Book of abstracts

Ministerstwo Edukacji i Nauki

# An eigenvalue problem for fully anisotropic elliptic operators

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**Abstract**

The existence of eigenfunctions for a class of fully anisotropic elliptic equations is established. The relevant equations are associated with constrained minimization problems for integral functionals depending on the gradient of competing functions through general anisotropic Young functions. In particular, the latter need neither be radial, nor have a polynomial growth, and are not even assumed to satisfy the so called $\Delta_2$-condition. In particular, our analysis requires the development of some new aspects of the theory of anisotropic Orlicz-Sobolev spaces. This is a joint work with G. di Blasio and F. Feo [1].

[1] A Alberico, G di Blasio & F Feo, An eigenvalue problem for the anisotropic $\Phi-$Laplacian, *Journal of Differential Equations*, **269** (2020), 4853–4883.

# Nonlinear elliptic critical problems in $\mathbb{R}^N$

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**Abstract**

In this talk we discuss some recent results concerning elliptic problems of $(p, q)$-Laplacian type in all of $\mathbb{R}^N$ with a nonlinearity involving both a critical term and a subcritical term, nonnegative nontrivial weights and a positive real parameter $\lambda$. In particular, under suitable conditions on the exponents of the nonlinearity, we obtain existence and multiplicity results with negative and positive energy depending on the range of the parameter $\lambda$, overcaming the double loss of compactness due both to the critical Sobolev's exponent $p^*$ and to the unboundedness of the domain. We analyze also the case of nonnegative nontrivial weights satisfying some symmetry conditions with respect to a certain group $T \subset O(N)$, where $O(N)$ is the group of orthogonal linear transformations in $\mathbb{R}^N$. Our proofs use variational methods and the concentration-compactness principle.

# Convergence of Non-autonomous Attractors for Subquintic Weakly Damped Wave Equation

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**Abstract**

We will present results about non-autonomous weakly damped wave equation with subquintic growth condition on the nonlinearity, which are based on Strichars estimations. We will discuss the existence and smoothness of pullback, uniform, and cocycle attractors and the relations between them. We will also see that these non-autonomous attractors converge upper-semicontinuously to the global attractor for the limit autonomous problem if the time-dependent nonlinearity tends to time independent function in an appropriate way.

## References

[1] Jakub Banaśkiewicz and Piotr Kalita. "Convergence of Non-autonomous Attractors for Subquintic Weakly Damped Wave Equation". In: *Applied Mathematics Optimization* (June 2021). DOI: 10.1007/s00245-021-09790-8.

[2] Matthew Blair, Hart Smith, and Christopher Sogge. "Strichartz estimates for the wave equation on manifolds with boundary". In: *Annales de l'Institut Henri Poincare (C) Non Linear Analysis* 26 (Sept. 2009), pp. 1817–1829. DOI: 10.1016/j.anihpc.2008.12.004.

[3] Varga Kalantarov, Anton Savostianov, and Sergey Zelik. "Attractors for Damped Quintic Wave Equations in Bounded Domains". In: *Annales Henri Poincaré* 17 (Sept. 2013). DOI: 10.1007/s00023-016-0480-y.

# Normalized ground states of the nonlinear Schrödinger equation with at least mass critical growth

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**Abstract**

We propose a simple minimization method to show the existence of least energy solutions to the normalized problem

$$\begin{cases} -\Delta u + \lambda u = g(u) \quad \text{in } \mathbb{R}^N, \ N \geq 3, \\ u \in H^1(\mathbb{R}^N), \\ \int_{\mathbb{R}^N} |u|^2 \, dx = \rho > 0, \end{cases}$$

where $\rho$ is prescribed and $(\lambda, u) \in \mathbb{R} \times H^1(\mathbb{R}^N)$ is to be determined. The new approach based on the direct minimization of the energy functional on the linear combination of Nehari and Pohozaev constraints intersected with the closed ball in $L^2(\mathbb{R}^N)$ of radius $\rho$ is demonstrated, which allows to provide general growth assumptions imposed on $g$. We cover the most known physical examples and nonlinearities with growth considered in the literature so far as well as we admit the mass critical growth at 0.

# Forward self-similar solutions of the Keller–Segel model in higher dimensions

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**Abstract**

We construct radial self-similar solutions of the, so called, minimal parabolic-elliptic Keller–Segel model in several space dimensions with radial, nonnegative initial conditions with are below the Chandrasekhar solution — the singular stationary solution of this system.
arXiv:2021.02571

# Nonlocal Douglas identity in $L^p$

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**Abstract**

We will discuss Hardy-Stein and Douglas identities for specific nonlinear nonlocal Sobolev-Bregman integral forms with unimodal Lévy measures. We will also explain that the corresponding Poisson integral defines an extension operator for the Sobolev-Bregman spaces. The results generalizes to the setting of $L^p$ spaces earlier results of the authors obtained for the (quadratic) Dirichlet forms and $L^2$ spaces. Here is the preprint.

# Uniformly Accurate Low Regularity Integrators For the Klein–Gordon Equation

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**Abstract**

We propose a novel class of uniformly accurate integrators for the Klein–Gordon equation which capture classical $c = 1$ as well as highly-oscillatory non-relativistic regimes $c \gg 1$ and, at the same time, allow for low regularity approximations. In particular, the schemes converge with order $\tau$ and $\tau^2$, respectively, under lower regularity assumptions than classical schemes, such as splitting or exponential integrator methods, require. The new schemes in addition preserve the nonlinear Schrödinger (NLS) limit on the discrete level. More precisely, we will design our schemes in such a way that in the limit $c \to \infty$ they converge to a recently introduced class of low regularity integrators for NLS.

# On certain estimates for a divergence form second order elliptic operator with unbounded coefficients

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**Abstract**

Suppose that $\mathcal{L}$ is a divergence form differential operator of the form $\mathcal{L}f(x) := e^{U(x)}/2\nabla_x \cdot \Big[e^{-U(x)}(I + H(x))\nabla_x f(x)\Big]$, where $U(x)$ is scalar valued and $H(x)$ is an anti-symmetric matrix valued function. We assume that they are $C^2$ regular but need not be bounded. We show that if $Z = \int_{\mathbb{R}^d} e^{-U(x)}dx < +\infty$ and there exists $\gamma_0 > 0$ such that $\int_{\mathbb{R}^d} e^{\gamma_0 \mathfrak{u}(x)\vee 0}\mu(dx) < +\infty$, where $\mathfrak{u}(x)$ is the supremum of the numerical range of matrix $-\nabla_x^2 U(x) + \nabla_x(\nabla_x \cdot H(x))$, then for any $1 \le p < q < +\infty$ we have $\|\nabla_x f\|_{L^p(\mu)} \le C\Big(\|\mathcal{L}f\|_{L^q(\mu)} + \|f\|_{L^q(\mu)}\Big)$ for $f \in C_0^\infty(\mathbb{R}^d)$. Here $d\mu = Z^{-1}e^{-U}dx$ and constant $C$ depends only on $p, q$, the dimension $d$ and $\gamma_0$. In addition, we give estimates on the spatial gradient of a semigroup $(P_t)_{t\ge 0}$ that corresponds to $\mathcal{L}$. Namely, there exist $C, t_* > 0$, depending only on $p, q$, the dimension $d$ and $\gamma_0$, such that $\|\nabla_x P_t f\|_{L^p(\mu)} \le C(t \wedge t_*)^{-1/2}\left\|P_{(t-t_*)_+} f\right\|_{L^q(\mu)}$, $t > 0$.

# Parabolic equations on low-dimensional structures

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**Abstract**

In many applications, a need to consider certain partial differential equations on non-regular, low-dimensional structures embedded in the Euclidean space arises naturally. For example, in engineering, it is sometimes necessary to analyze a transfer of heat in conductors of umbrella-like shape or in objects of similar type, which are far from being a manifold in a classical sense. Different attempts to variational problems on low-dimensional subsets of the Euclidean space are already known. A major disadvantage of such methods is that their applicability is limited by the regularity of the structures – they cannot be applied to many interesting classes of domains. It is important to establish a consistent theory that will cover also less regular cases. Recently, a paper has been published, in which the authors establish the weak form of the stationary heat equation on a very wide class of structures, which are called multijunction measures.

The goal of my presentation is to showcase my recent results on the theory of parabolic equations on lower-dimensional subsets of $\mathbb{R}^n$. First, I am going to briefly describe how to define a second-order operator in the related setting and I am going to introduce a proper generalization of Sobolev spaces. The main part of the talk is going to be focused on the existential results, including the approach via the semigroup theory. The theorems which I am going to discuss are quite general, but it turns out that on structures of different regularity we need to realize certain proofs in an alternative way. I am going to highlight differences between structures on which the Poincaré inequality is valid and less regular ones. Besides that, I also plan to present the results on higher regularity of solutions to elliptic problems; that is, I am going to explain a correspondence between weak Sobolev solutions and those which belong to higher-order spaces.

# Numerical approximations of nonlocal Mean Field Games

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joint work with Olav Ersland and Espen R. Jakobsen
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**Abstract**

I will discuss the numerical approximation of a class of mean field game equations (coupled system of PDEs consisting Hamilton-Jacobi-Bellman and Fokker-Planck equations) with *nonlocal diffusion*. The problem may include degenerate diffusion and our scheme is based on semi-Lagrangian approximation of the underlying control problem/ games. The prescribed schemes are monotone, stable and consistent. I will discuss the convergence results along subsequences for degenerate equations in one space dimension and also for nondegenerate equations in arbitrary dimensions.

# Nonuniqueness of solutions to the Euler equations with vorticity in a Lorentz space

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**Abstract**

For the two dimensional Euler equations, a classical result by Yudovich states that solutions are unique in the class of bounded vorticity; it is a celebrated open problem whether this uniqueness result can be extended in other integrability spaces. We prove in this note that such uniqueness theorem fails in the class of vector fields $u$ with uniformly bounded kinetic energy and vorticity in the Lorentz space $L^{1,\infty}$.

# Stationary Euler flows near the Kolmogorov flow

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**Abstract**

We exhibit a large family of new, non-trivial stationary states of analytic regularity, that are arbitrarily close to the Kolmogorov flow on the square torus. Our construction of these stationary states builds on a degeneracy in the global structure of the Kolmogorov flow. This has surprising consequences in the context of inviscid damping in 2D Euler and enhanced dissipation in Navier-Stokes.

# Discontinuous stationary solutions to reaction-diffusion-ODE systems

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**Abstract**

In this talk I will present a construction of discontinuous stationary solutions to general reaction-diffusion-ODE systems as well as a sufficient conditions for their stability. This is a joint work with Anna Marciniak-Czochra, Grzegorz Karch and Kanako Suzuki.

These results are contained in our works:
– S. Cygan, A. Marciniak-Czochra, G. Karch, and K. Suzuki, *Instability of all regular stationary solutions to reaction-diffusion-ODE systems*, preprint (2021), arXiv:2105.05023.
– S. Cygan, A. Marciniak-Czochra, G. Karch, and K. Suzuki, *Stable discontinuous stationary solutions to reaction-diffusion-ODE systems*, preprint (2021).

# Attractors for Dissipative Fourth Order Problems in $\mathbb{R}^N$

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**Abstract**

We consider a Cauchy problem for a dissipative fourth order parabolic equation in $\mathbb{R}^N$ with a general potential. Using the quasi-stability method by Chueshov and Lasiecka we estimate from above fractal dimension of a global attractor. We also show that it is contained in a finite dimensional exponential attractor. This is a joint work with Jan W. Cholewa (University of Silesia in Katowice) based on the article

Jan W. Cholewa, R. Czaja, On fractal dimension of global and exponential attractors for dissipative higher order parabolic problems in $\mathbb{R}^N$ with general potential, in *Contemporary Approaches and Methods in Fundamental Mathematics and Mechanics*, Understanding Complex Systems, Victor A. Sadovnichiy, Michael Z. Zgurovsky (Eds.), Springer, 2021, pp. 293-314.

# Boundary strong unique continuation property for fractional elliptic equations

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**Abstract**

I will present a recent result of strong unique continuation property at boundary points for solutions to the following fractional elliptic equation

$$(-\Delta)^s u = hu$$

in a bounded domain contained in $\mathbb{R}^N$, $N \geq 2$, $s \in (0,1)$, under some outer homogeneous Dirichlet boundary condition.

The idea is to consider the Caffarelli-Silvestre extension, thus providing an equivalent formulation of the fractional equation as a local problem in one dimension more. Then, after constructing a procedure of approximation of the domain, in the local context the classical approach developed by Garofalo and Lin allows to derive unique continuation from doubling conditions as a consequence of the boundedness of a suitable Almgren type frequency function.

Combining the aforementioned analysis with blow-up arguments, a strong unique continuation can be achieved in the nonlocal setting.

# The Bernstein technique for integrodifferential equations

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**Abstract**

We present a version of the classical Bernstein technique for integro-differential operators. We provide first and one-sided second derivative estimates for solutions to fractional equations, including some convex fully nonlinear equations of order smaller than two, for which we prove uniform estimates as their order approaches two. Our method is robust enough to be applied to some Pucci-type extremal equations and to obstacle problems for fractional operators, although several of the results are new even in the linear case. The result discussed come from a joint work with Xavier Cabré and Enrico Valdinoci.

# On the relevance of stochastic models in turbulence

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**Abstract**

We consider statistical limit of solution to the compressible Navier–Stokes system in the high Reynolds number regime in a domain exterior to a rigid body. We investigate to what extent this highly turbulent regime can be modelled by an external stochastic perturbation, as suggested in the related physics literature. To this end, we interpret the statistical limit as a stochastic process on the associated trajectory space. We suppose that the limit process is statistically equivalent to a solution of the stochastic compressible Euler system. Then, necessarily,

- the stochastic forcing is not active – the limit is a statistical solution of the deterministic Euler system;
- the solutions S–converge to the limit;
- if, in addition, the expected value of the limit process solves the Euler system, then the limit is deterministic and the convergence is strong in the $L^p$–sense.

These results strongly indicate that a stochastic forcing may not be a suitable model for turbulent randomness in compressible fluid flows.

# Geometric aspects of the 1-Laplacian

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**Abstract**

The Dirichlet problem for the 1-Laplacian operator is the degenerate elliptic equation

$$-\mathrm{div}\left(\frac{Du}{|Du|}\right) = 0, \qquad u|_{\partial\Omega} = g \in L^1(\partial\Omega). \tag{1}$$

It is typically formulated as the *least gradient problem*

$$\min\left\{ \int_\Omega |Du| : \, u \in BV(\Omega), \, u|_{\partial\Omega} = g \in L^1(\partial\Omega) \right\}. \tag{2}$$

Note that equation (1) implies that the mean curvature of the level sets of a solution $u$ vanishes. Therefore, the 1-Laplace equation is linked to the study of minimal surfaces, but also (among others) to conductivity imaging, shape optimisation, and two-dimensional optimal transport. In this talk, we focus on the relations to minimal surfaces and optimal transport in 2D to highlight the geometry behind the 1-Laplace operator. We also present the applications of methods from geometric measure theory and optimal transport to existence, regularity, and stability of functions of least gradient (i.e., solutions to (2)).

# Optimal Hardy inequalities for the fractional Laplacian on $L^p$

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**Abstract**

Let $d \geq 1$ and $0 < \alpha < d \wedge 2$. For $p \in (1, \infty)$ and $u : \mathbb{R}^d \to \mathbb{R}$ we define the $p$-form,

$$\mathcal{E}_p[u] := \frac{1}{2} \int_{\mathbb{R}^d} \int_{\mathbb{R}^d} (u(x) - u(y))(u(x)^{\langle p-1 \rangle} - u(y)^{\langle p-1 \rangle})\nu(x - y)\, dy\, dx,$$

where

$$\nu(z) = \frac{2^\alpha \Gamma\big((d + \alpha)/2\big)\pi^{-d/2}}{|\Gamma(-\alpha/2)|} |z|^{-d-\alpha}, \quad z \in \mathbb{R}^d,$$

and $a^{\langle k \rangle} := |a|^k \operatorname{sgn} a$. During the talk I will discuss the following inequality

$$\mathcal{E}_p[u] \geq C \int_{\mathbb{R}^d} \frac{|u(x)|^p}{|x|^\alpha}\, dx, \quad u \in L^p(\mathbb{R}^d).$$

The explicit formula for the best constant $C$ will be given. The talk will be based on the recent paper `https://arxiv.org/abs/2103.06550`

# Existence of strong solutions to a class of compressible non-Newtonian Navier-Stokes equations

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**Abstract**

We discuss a local-in-time existence of a strong solution to the generalized compressible Navier-Stokes equation for arbitrarily large initial data. The existence of the solution is obtained by the maximal $L^p - L^q$-regularity theorem for linearized equations which is proven with help of the Weis multiplier theorem. The result can be seen as generalization of the work of Shibata and Enomoto (devoted to compressible fluids) to compressible non-Newtonian fluids.

# Singular behavior of solutions to reaction-diffusion-ODE systems

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**Abstract**

I shall review results, obtained jointly with Anna Marciniak-Czochra, Kanako Suzuki, and Szymon Cygan, on a certain class of reaction-diffusion systems from Mathematical Biology, where ordinary differential equations are coupled with one reaction-diffusion equation. Such systems may have regular (*i.e.* sufficiently smooth) stationary solutions, however, all of them are unstable. We showed that solutions to initial-boundary value problems for such reaction-diffusion-ODE systems may behave in a singular way for large values of time which means that they may blowup in a finite time or converge towards discontinuous stationary solutions.

These results are contained in our works:

– A. Marciniak-Czochra, G. Karch, K. Suzuki, *Unstable patterns in reaction-diffusion model of early carcinogenesis.* J. Math. Pures Appl. (9) 99 (2013), no. 5, 509–543.

– A. Marciniak-Czochra, G. Karch, K. Suzuki, J. Zienkiewicz, *Diffusion-driven blowup of nonnegative solutions to reaction-diffusion-ODE systems.* Differential Integral Equations 29 (2016), 715–730.

– A. Marciniak-Czochra, G. Karch, and K. Suzuki, *Instability of Turing patterns in reaction-diffusion-ODE systems*, J. Math. Biol. 74 (2017), 583–618.

– S. Cygan, A. Marciniak-Czochra, G. Karch, and K. Suzuki, *Instability of all regular stationary solutions to reaction-diffusion-ODE systems*, preprint (2021), arXiv:2105.05023.

– S. Cygan, A. Marciniak-Czochra, G. Karch, and K. Suzuki, *Stable discontinuous stationary solutions to reaction-diffusion-ODE systems*, preprint (2021).

# Elliptic (non-local) PDEs with singular data - Brezis' theory of reduced measures

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**Abstract**

Let $E$ be a locally compact separable metric space and $m$ be a Radon measure on $E$ with full support. The presentation is devoted to the existence problem for the following equation

$$-Au + Vu = f(\cdot, u) + \mu, \tag{1}$$

where $A$ is a self-adjoint linear operator on $L^2(E;m)$ generating a Markov semigroup, $V$ is a locally (quasi-)integrable non-negative function on $E$, $f : E \times \mathbb{R} \to \mathbb{R}$ is a real Carathéodory function satisfying the sign condition, i.e.

$$f(x, y) \cdot y \leq 0, \quad x \in E,\, y \in \mathbb{R},$$

and $\mu$ is a Borel measure on $E$. The model example of a local operator which fits into our framework is uniformly elliptic divergence form diffusion operator

$$Au = \sum_{i,j=1}^{d} (a_{i,j} u_{x_i})_{x_j},$$

whereas a model example of a non-local operator fitting into the framework is the fractional Laplacian

$$Au = \Delta^\alpha u(x) = c_\alpha \lim_{\varepsilon \searrow 0} \int_{\mathbb{R}^d \setminus B(0,\varepsilon)} \frac{u(y) - u(x)}{|x - y|^{d+2\alpha}}\, dy$$

for $\alpha \in (0, 1)$. It is well known that the mechanism of existence and non-existence of a solution, hidden in the equation (1), is very subtle and sensitive to the change of data. In case $f \equiv 0$, we will give necessary and sufficient condition for the existence of a solution to (1). Whereas, in case $V \equiv 0$, we will provide, using Brezis' theory of reduced measures (see [1]–[3]), a characterization of the class of Borel measures for which (1) is solvable. We will end the presentation with a discussion on the Chern-Simons equation

$$-\Delta u = e^u (1 - e^u) + \mu$$

and its variants (see [4]).

[1] Brezis, H., Marcus, M., Ponce, A.C.: A new concept of reduced measure for nonlinear elliptic equations. *C. R. Math. Acad. Sci.* **339**, 169–174 (2004)
[2] Brezis, H., Marcus, M., Ponce, A.C.: Nonlinear elliptic equations with measures revisited. In: Mathematical Aspects of Nonlinear Dispersive Equations (J. Bourgain, C. Kenig, S. Klainerman, eds.), Annals of Mathematics Studies, **163**, Princeton University Press, Princeton, NJ, 55–110 (2007)
[3] Klimsiak, T.: Reduced measures for semilinear elliptic equations involving Dirichlet operators. *Calc. Var.* 55:78 (2016)
[4] Ponce, A.C., Presoto, A.E.: Limit solutions of the Chern-Simons equation. *Nonlinear Anal.* **84** (2013) 91–102.

# Nonlinear equations with generalized fractional Laplacian

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**Abstract**

We study a generalization of the power of Laplace operator with null Dirichlet conditions by means of the spectral theory and prove several existence results for nonlinear equations with such operators. We consider two problems with generalized fractional Laplacian. Firstly, we consider the following equation

$$g(-\Delta)u = f(x,u),$$

where $g(-\Delta)$ is generalized fractional Laplacian. Next, we investigate heat equation with generalized fractional Laplacian

$$u_t + g(-\Delta)u = f(t,x,u).$$

# Measure-valued solutions and weak-strong uniqueness for the incompressible inviscid fluid-rigid body interaction

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**Abstract**

We consider a coupled system of partial and ordinary differential equations describing the interaction between an incompressible inviscid fluid and a rigid body moving freely inside the fluid. We prove the existence of measure-valued solutions which is generated by the vanishing viscosity limit of incompressible fluid-rigid body interaction system under some physically constitutive relations. Moreover, we show that the measure-valued solution coincides with strong solution on the interval of its existence. This relies on the weak-strong uniqueness analysis. This is the first result of an existence of measure-valued solution and weak-strong uniqueness in measure-valued sense in the case of inviscid fluid-structure interaction.

# Fully nonlinear parabolic mean field games, local and nonlocal

Miłosz Krupski
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joint work with Indranil Chowdhury & Espen R. Jakobsen
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**Abstract**

A *mean field game* is a special system of PDEs consising of a Hamilton–Jacobi and a Fokker–Planck equation. In this talk I will introduce a class of fully nonlinear mean field games posed in $[0, T] \times \mathbb{R}^d$ derived from a control problem for local or nonlocal diffusions (Lévy processes).

Under some abstract assumptions, existence and uniqueness of solutions can be established. I will discuss two special cases: strongly degenerate equations of order less than one, and a class of nondegenerate equations.

# Flow invariance of closed sets

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**Abstract**

I shall discuss the criteria for the viablity, the invariance and the strong invariance of level sets of locally Lipschitz functionals (i.e. in particular arbitrary closed sets) with respect to the flow generated by an infinite dimensional nonlinear evolution problem of the form $u_t + \mathbf{A}u = f(t, u)$, where $\mathbf{A}$ is a quasi $m$-accretive (nonlinear or even multivalued, in general) operator acting in a reflexive Banach space $f$ is continuous. Some concrete applications will illustrate the meaning of the discussed results.

# Proof of the pitchfork bifurcation in the Kuramoto-Sivashinsky equation.

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**Abstract**

We will sketch proof of the pitchfork bifurcation for $\mu = 1$ in the Kuramoto-Sivashinsky equation

$$u_t = (u^2)_x - u_{xx} - \mu u_{xxxx}$$

with odd and periodic boundary conditions. More exactly, we will discuss how to show that the fixed point $u = 0$ is hyperbolic for $\mu \neq 1$ close to 1 and that after bifurcation two hyperbolic fixed points are born, to which there are heteroclinic connections from the origin. In proof unstable manifold theorem and normal forms methods are used.

# Effective high order integrators for linear Klein-Gordon equations in low to highly oscillatory regimes

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**Abstract**

We consider the linear Klein-Gordon equation

$$\partial_t^2 \psi(\mathbf{x},t) \quad = \quad \Delta\psi(\mathbf{x},t) + f(\mathbf{x},t)\psi(\mathbf{x},t) \tag{1}$$

with initial condition $\psi(x,0) = \psi_0(x)$, $\psi'(x,0) = \psi_0'(x)$ equipped with periodic boundary conditions (that is $x \in \mathbb{T}^d$). Here, $f(\mathbf{x},t)$ is a given, periodic in space, under the form

$$f(\mathbf{x},t) \quad = \quad \alpha(\mathbf{x},t) + \sum_n a_n(\mathbf{x},t)\mathrm{e}^{\mathrm{i}\omega n t}, \tag{2}$$

where $\omega \gg 1$ are the frequencies of the osculations. Function $\alpha(\mathbf{x},t)$ is non-oscillatory and $\sum_n a_n(\mathbf{x},t)\mathrm{e}^{\mathrm{i}\omega n t}$ is purely oscillatory function.

I will present three computational approaches to the problem:

1. Modulated Fourier based approach performing asymptotic behavior in frequencies $\omega$;
2. Splitting methodologies resulting in second order convergence $\mathcal{O}((\Delta t)^2)$ holding uniformly in $\omega$ and fourth order convergence $\mathcal{O}((\Delta t)^4)$ under the scaling $\Delta t \gtrsim 1/\sqrt[3]{|\omega|}$;
3. Duhamel integrators of third order $\mathcal{O}((\Delta t)^3)$ uniformly in $\omega$.

I will briefly discuss the derivation of the methods, error analysis and present plenty of numerical examples comparing derived methods against each other and against other methods known from the literature.

Results obtain in joined work with K. Kropielnicka ( Institute of Mathematics, Polish Academy of Sciences), K. Schratz (Laboratoire Jacques-Louis Lions, Sorbonne Université, France), R. Perczyński, (University of Gdańsk, Faculty of Mathematics, Physics and Informatics, Poland) and M. Condon, (School of Electronic Engineering, Glasnevin, Ireland).

# Connecting random fields on manifolds and stochastic partial differential equations in simulations

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**Abstract**

Random fields on manifolds can be used as building blocks for solutions to stochastic partial differential equations or they can be described by stochastic partial differential equations. In this talk I present recent developments in numerical approximations of random fields and solutions to stochastic partial differential equations on manifolds and connect the two. More specifically, we look at the stochastic wave equation on the sphere and approximations of Gaussian random fields on manifolds using suitable finite element methods. Throughout the talk, theory and convergence analysis are combined with numerical examples and simulations.

This talk is based on joint work with David Cohen, Erik Jansson, Mihly Kovcs, and Mike Pereira.

# Attractors as information structures

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**Abstract**

We will present main results on the topological and geometrical description of global attractors for infinite-dimensional dynamical systems. In particular, the continuity of attractors, their gradient structure and their Morse-Smale characterization. We will also show some results on their robustness under autonomous and non-autonomous perturbation. These approach leads to the analysis of attractors as information structures, which are being applied to Ecology and Neuroscience phenomena in order to describe structural stability in mutualistic system and conscious states for subjects with severe brain damage.

# Long time behavior in a flow-structure interaction

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**Abstract**

Flow-structure interactions are ubiquitous in nature. Problems such as attenuation of turbulence or flutter in an oscillating structure [Tacoma bridge] are prime examples of relevant applications. Mathematically, the models are represented by a 3 D Euler Equation coupled to a nonlinear dynamic elasticity on a 2 D manifold. Strong boundary-type coupling at the interface between the two media is at the center of the analysis. This provides for a rich mathematical structure, opening the door to several unresolved problems in the area of nonlinear PDEs, dynamical systems and related harmonic analysis and geometry. This talk aims at providing a brief overview of recent developments in the area along with a presentation of some recent advances addressing the issues of control and long time behavior [partial structural attractors] subject to mixed boundary conditions arising in modeling of the interface between the two environments.

# An efficient method to approximate wave packets on the whole real line

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**Abstract**

Highly oscillatory wave packets play an important role in quantum mechanics especially in the semi-classically scaled time dependent Schrödinger equation. We compare three different orthogonal system in $L_2(\mathbb{R})$ which can be used to build a spectral method for solving such problems. These systems all have banded skew-Hermitian differentiation matrices, which implies that they will respect the Born interpretation of quantum mechanics, and the linear algebra of the spectral method is simplified. We show in the high-frequency regime, that the Malmquist–Takenaka basis is superior, in a practical sense, to the more commonly used Hermite functions and Stretched Fourier expansions for approximating wave packets. We also show that Malmquist–Takenaka expansions converge exponentially fast to wave packets, which goes against the established theory on this basis. The main body of the talk will look at our proof which uses the method of steepest descent in the complex plane. This is joint work with Arieh Iserles (Cambridge) and Marcus Webb (Manchester).

# Existence of the 1-harmonic map flow


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**Abstract**

Similarly as in the real-valued case, the total variation of maps taking values in a Riemannian manifold $\mathcal{N}$, defined by

$$\int_\Omega |\nabla u|$$

for $u \in C^1(\Omega, \mathcal{N})$, extends to a lower semicontinuous functional $L^2(\Omega, \mathcal{N}) \to [0, +\infty]$. However, in general this functional is not geodesically semiconvex, so the existence of its gradient flow is not provided by general variational theory. Alternatively, one can try to apply the theory of parabolic PDE systems, mimicking the approach used for $p$-harmonic map flows, $p > 1$. This poses some difficulties, because the PDE system corresponding to the flow is strongly nonlinear, singular and degenerate. However, in some cases, this approach was successful. In this talk, I will report on recent results on the existence of the flow obtained in cooperation with L. Giacomelli and S. Moll.

# Highly oscillatory quadrature exploiting banded representations of differential operators

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**Abstract**

In this talk, we will study the efficient approximation of highly oscillatory integrals using so-called Filon and Levin methods. These are amongst the most successful methods for approximating such integrals, a task that becomes prohibitively expensive for classical quadrature.

However, Filon and Levin methods are very much topics of active research and have inherent limitations that are yet to be overcome. Perhaps a somewhat surprising connection can be made between the constructions of these methods and differential operators with banded matrix representations with respect to relevant interpolation bases.

We exploit this connection to address the important 'moment-problem' in Filon methods and hence to treat integrals arising in wave scattering that were previously not attainable in the Filon framework. Moreover, we exploit this connection to improve the efficiency of computing the Levin method, reducing the cost of approximation from $\mathcal{O}(\nu^3)$ to just $\mathcal{O}(\nu \log \nu)$ in the number of quadrature points $\nu$. This is joint work with Arieh Iserles.

# $p$-Hierarchical Enrichment of Eigenvalue/Eigenvector Approximations

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**Abstract**

Many real life problems lead to challenging PDE eigenvalue problems, e.g., vibrations of structures or calculation of energy levels in quantum mechanics. A lot of research is devoted to the so-called Adaptive Finite Element Method (AFEM) which allows discretization of the governing PDE, solving the finite dimensional algebraic eigenvalue problem and iteratively improving obtained numerical approximations.

Investigating the behavior of the AFEM eigenvalue/eigenvector algorithms from the point of view of numerical linear algebra (NLA), is the scope of intensive research in the last few years. Several theoretical as well as algorithmic results clearly indicate a real necessity of engaging various NLA techniques into numerical PDE solvers, not only to obtain meaningful and relevant solutions of the real-world problems, but also to encourage the transition from hardware to algorithm oriented computational techniques. In this work, we show that an application of just one implicit inverse iteration step on the computed FEM-Ritz vector not only yields a super-converging Ritz value, but also significantly reduces the cost of underlying finite element computations. We will illustrate the ability of the presented method to solve both selfadjoint and nonselfadjoint PDE eigenvalue problems up to the accuracy guaranteed by the higher order finite elements while keeping the computational cost of the lower finite elements approximation, i.e., obtaining approximations of the $\mathbb{P}_2$ finite elements accuracy within the cost of $\mathbb{P}_1$ finite elements computations.

# Perturbations beyond Schauder

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**Abstract**

Nonuniformly elliptic and parabolic problems are a classical topic since the pioneering work of Finn, Gilbarg, Ladyzhenskaya, Uraltseva, Trudinger, Serrin, Ivanov, Ivochkina. There has been an intensive activity in proving regularity results for solutions since the 60s. Such problems play also an important role in the Calculus of Variations, where they arise in connection to the Euler-Lagrange equations of functionals with non-standard growth conditions. A basic issue that has remained open is the validity of Schauder estimates, whose lack is connected with the failure of classical perturbation methods in the nonuniformly elliptic case. In this talk I will present the first solution to this problem, providing an approach to Hölder continuity which is alternative to the classical ones based on perturbations.

From joint work with Cristiana De Filippis (Torino)

# Stability of singularities of minimizing harmonic maps

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**Abstract**

Minimizing harmonic maps – i.e., maps into a fixed manifold that minimize the Dirichlet energy – are known to be smooth outside a singular set of codimension 3. Here, we consider maps into the standard sphere $\mathbb{S}^2$ and investigate how the singular set is affected by small perturbations of the prescribed boundary map. We show a simple stability result in which the singularities of two minimizing maps are compared using the Wasserstein distance. The talk is based on joint work with Katarzyna Mazowiecka and Armin Schikorra.

# Solutions to the Monge-Ampère equation with polyhedral and Y-shaped singularities

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**Abstract**

Abstract: We will discuss examples of functions that that solve the Monge-Ampère equation $\det(D^2u) = 1$ away from finitely many points, and exhibit polyhedral and Y-shaped singularities. Along the way we will discuss geometric and applied motivations for constructing such examples, as well as their connection to a certain obstacle problem.

# Long-time behavior of discrete volume-preserving mean curvature flat flows

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**Abstract**

We consider the Almgren-Taylor-Wang implicit time-discretization scheme for the volume-preserving mean curvature flow in $\mathbb{R}^N$ and study its long-time behavior for arbitrary bounded initial sets of finite perimeter. Open problems and generalization will be discussed. This work is in collaboration with M. Ponsiglione and E. Spadaro.

# Burgers equation on graphs

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**Abstract**

Consider the classical inviscid Burgers equation $u_t + uu_x = 0$ in a monodimensional case. The basic interpretation of the system explains the motion of one wave, creation of shocks and rarefaction waves. However the theory does not capture the interaction of waves. Big waves eat smaller one. We can not obtain a passing through phenomenon between waves.

Our idea is to extend the monodimensional structure of the domain to a graph, giving the possibility for the solution to take different paths. This model is a motivation to introduce a theory of the Burgers equation on metric graphs. The crucial point is the behavior of solutions at vertexes, which must be suitably determined. I will show some interesting examples as well as key points of mathematical theory.

The talk is based on the results jointly with Aleksandra Puchalska (Warszawa).

# On the problem of the motion of a rigid body in a compressible fluid

Šárka Nečasová, Arnab Roy
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Mythily Ramaswamy
NASI Senior Scientist, ICTS-TIFR (India)

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**Abstract**

We study a 3D nonlinear moving boundary fluid-structure interaction problem describing the interaction of the fluid flow with a rigid body. The fluid flow is governed by 3D compressible Navier-Stokes equations, while the motion of the rigid body is described by a system of ordinary differential equations called Euler equations for the rigid body. The equations are fully coupled via dynamical and kinematic coupling conditions. We consider the Navier-slip boundary condition at the interface as well as at the boundary of the domain and we show existence of a weak solution of the fluid-structure system up to collision.

# Elliptic equations in Musielak-Orlicz spaces

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**Abstract**

We deal with the nonlinear elliptic equations in Musielak–Orlicz spaces equipped with the convex modular. We do not assume that it is $N$-function. We do not use any growth conditions on our non-linearity. We look for positive weak solutions. We use developed here some generalization of Krasnosielskii fixed point theorem on a cone, in modular space.

# A quantitative fractional isocapacitary inequality

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**Abstract**

In view of the fractional generalization of the Pólya-Szegö inequality, it is known that, sharing the same amount of Lebesgue measure, balls minimize the fractional (Newtonian) capacity. I would like to present a stability inequality which quantifies the deviation of the fractional capacity of a set from the capacity of the ball with the same volume. In particular, the measure of such variation is a notion of asymmetry of the set with respect to a ball. This is a joint work with E. Cinti and B. Ruffini.

# Weak solutions for the Norton-Hoff model with full temperature dependence

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**Abstract**

The presentation deals with the existence of weak solution for a quasi-static evolution of thermo-visco-elastic model with Norton-Hoff law of plasticity. The dependence on temperature occurs both in the elastic constitutive equations (generalised Hooke's law) and in describing the evolution of visco-elastic strain. These thermal effects have not been previously considered. The approximations of the considered models did not allow in literature such a general models. The main idea of proof is the revocation to R. Temam articles on the plasticity from eighties of the previous century and to write down the equations related to the plastic deformations in the same way. For the obtained equations we propose approximations in a flow rule. Thanks to this manner of writing the equations, we show the existence of a weak solution.

# Asymptotic expansions for PDEs with highly oscillatory input term

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**Abstract**

Let us consider a well posed, linear partial differential equation with highly oscillatory input term

$$\partial_t = \mathcal{L}u + f(x,t)u(x,t), \quad t \in [0,T], \quad x \in \Omega, \tag{1}$$
$$f(x,t) = \alpha(x)e^{i\omega t}, \quad \omega \gg 1.$$

Numerical approach based on presentation of the solution as Modulated Fourier transform

$$u(x,t) \sim \sum_{r=1}^{\infty} \frac{1}{\omega^r} \sum_{n=0}^{\infty} p_{r,n}(x,t)e^{in\omega t},$$

where coefficients $p_{n,r}(x,t)$ (independent of $\omega$) are computed numerically is a well-known and investigated tool in asymptotic numerical approach for this kind of problems. Although its efficiency has been recognised, the error analysis has not been investigated rigorously.

In this presentation I will provide analytical form of such an expansion (that is I derive analytically formulas for $p_{n,r}(x,t)$) and the rigorous form of the error term. It is the first step towards asymptotic expansions for PDEs with more general form of highly oscillatory input term

$$f(x,t) = \sum_{k=-N}^{N} \alpha_k(x,t)e^{ik\omega t}, \quad \omega \gg 1.$$

The presented results are obtained in collaboration with Karolina Kropielnicka, IM PAN.

# Numerical methods for nonlocal and nonlinear parabolic equations with applications in hydrology and climatology

Łukasz Płociniczak*,†

Many natural and industrial phenomena exhibit nonlocal behaviour in temporal or spatial dimension. The former is responsible for processes for which its whole history influences the present state. The latter, on the other hand, indicates that faraway regions of the domain may have some impact on local points. This is useful in describing media of high heterogeneity.

Partial differential equations that are nonlocal involve one or several integral operators that encode this behaviour. For example, Riemann-Liouville or Caputo derivatives are used in temporal direction, while fractional Laplacian or its relatives describe spatial nonlocality. When it comes to numerical methods the discretization of these requires more care than their classical versions. Moreover, it is usually much more expensive, both on CPU and the memory, to conduct simulations involving nonlocal equations.

In this talk we will present several approaches to discretize nonlocal and nonlinear parabolic equations. These include: transformation into a pure integral equation for the time-fractional porous medium equation and Galerkin spectral methods for a general parabolic equation with temporal nonlocality. We will prove stability and convergence of these methods illustrating all the theoretical results with numerical simulations implemented in Julia programming language with parallelization. The talk is based on [2, 3, 1, 4].

## Acknowledgement

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[4] Łukasz Płociniczak. A linear galerkin numerical method for a strongly nonlinear subdiffusion equation. *in preparation*, 2021.

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# Continuity equation and vacuum regions in compressible flows

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**Abstract**

We investigate the creation and properties of eventual vacuum regions in the weak solutions of the continuity equation, in general, and in the weak solutions of compressible Navier–Stokes equations, in particular. The main results are based on the analysis of renormalized solutions to the continuity and pure transport equations and their inter-relations. The presentation is based on the paper Novotný, Pokorný: Continuity equation and vacuum regions in compressible flows, accepted to J. Evol. Equ., https://doi.org/10.1007/s00028-021-00704-3.

# What is graph surgery? A short summary on spectral properties of linear evolution equations on a metric graphs

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**Abstract**

In the talk we focus on linear dynamical process, transport and diffusion, along the edges of a graph with appropriate transmission conditions in the vertices. The semigroup setting of a problem provoked a series of articles that joined the spectral properties of Laplacian on the metric graph with a spectrum of associated semigroup, and with the geometric structure of the underlying graph. In the talk we briefly present how the change of the geometry of a graph, known as *graph surgery*, can influence the spectrum. Then we present first observations on similar results for the operator of the first derivative on a metric graph.

## References

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[2] G. Berkolaiko, J. Kennedy, P. Kurasov, D. Mugnolo, Surgery principles for the spectral analysis of quantum graphs (2019) Transactions of the American Mathematical Society, vol. 372, 5153 – 5197.

# The Navier–Stokes equations on expanding periodic domains

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**Abstract**

Given a compactly supported initial velocity or initial vorticity, we show that the solutions of the Navier–Stokes equations on the periodic domains $[-L, L]^3$ converge strongly to the solution on the whole space $\mathbb{R}^3$ with the same initial data. This has implication for the 'transfer of regularity' from solutions on the whole space to solutions on large enough periodic domains.

# Existence of strong solutions for a compressible viscous fluid and a wave equation interaction system

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**Abstract**

In this talk, we consider a fluid-structure interaction system where the fluid is viscous and compressible and where the structure is a part of the boundary of the fluid domain and is deformable. The reference configuration for the fluid domain is a rectangular cuboid with the elastic structure being the top face. The fluid is governed by the barotropic compressible Navier–Stokes system, whereas the structure displacement is described by a wave equation. We show that the corresponding coupled system admits a unique, locally-in-time strong solution for an initial fluid density and an initial fluid velocity in $H^3$ and for an initial deformation and an initial deformation velocity in $H^4$ and $H^3$ respectively.

This is a joint work with Debayan Maity (TIFR-CAM, India) and Takéo Takahashi (INRIA-Nancy, France).

# A biased review of space-fractional diffusion problems

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**Abstract**

We study an anomalous diffusion operator with space fractional Caputo derivative, $D_x^\alpha$. The eq. takes the form,

$$\partial_t u - \partial_x D_x^\alpha u = 0$$

augmented with initial and boundary conditions.

The problem arises from modeling of flows in heterogeneous porous media. In principle it is a free boundary problem. We present existence and uniqueness results in a cylindrical domain from two different point of views. The first one is in the framework of viscosity solutions. The second one is uses the theory of analytical semigroups and eventually yields solutions to the free boundary problem.

We show continuous dependence of solutions on the fractional order of differentiation $\alpha$: when $\alpha \to 0$ we obtain a transport eq.; for $\alpha$ tending to 1 we get the heat equation. In this context we study the question of the speed of the signal propagation.

This is a report on projects with A.Kubica, T.Namba, K.Ryszewska, V.Voller.

# Infinitely many entire solutions to the curl-curl problem with critical exponent

Jacopo Schino

We prove the existence of an unbounded sequence of solutions to

$$\nabla \times \nabla \times \mathbf{U} = |\mathbf{U}|^4 \mathbf{U}, \quad \mathbf{U} \colon \mathbb{R}^3 \to \mathbb{R}^3. \tag{1}$$

We combine a first group action introduced in [1] to reduce (1) to the vector Yamabe problem with a second group action introduced in [2] to recover compactness in the critical case.

This is joint work with Michał Gaczkowski and Jarosław Mederski.

## References

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[2] W. Ding: *On a Conformally Invariant Elliptic Equation on* $\mathbb{R}^n$, Comm. Math. Phys., 107, 331–335, (1986).

# On the regularity theory for non-uniformly elliptic problems

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**Abstract**

I will discuss regularity properties for solutions of linear second order non-uniformly elliptic equations in divergence form. Assuming certain integrability conditions on the coefficient field, we obtain local boundedness and validity of Harnack inequality. The assumed integrability assumptions are sharp and improve upon classical results due to Trudinger from the 1970s.

If time permits I will discuss applications to nonlinear non-uniformly elliptic problems and/or homogenization theory.

The talk is mainly based on joint works with Peter Bella.

# Optimal Liouville theorem for a semilinear Ornstein-Uhlenbeck equation

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**Abstract**

In their seminal 1985 paper Giga and Kohn analysed the blow-up behaviour of the subcritical Fujita equation through a Liouville theorem for an associated elliptic equation of Ornstein-Uhlenbeck type. In a subsquent work Giga provided a conditional extension of this Liouville theorem to a natural broader class of semilinear Ornstein-Uhlenbeck equations and posed a question of its unconditional validity, i.e. in the class of bounded entire solutions. In this talk we will show that indeed such a result holds. The key ingredient of the demonstration relies on a generalisation of the Rellich-Pohozaev type argument which employs a family of special multipliers based on Kummer functions.

# Fast reaction limit with nonmonotone reaction function

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**Abstract**

I will discuss two recent papers [1, 2] on the reaction-diffusion system

$$\partial_t u^\varepsilon = \frac{v^\varepsilon - F(u^\varepsilon)}{\varepsilon},$$
$$\partial_t v^\varepsilon = \Delta v^\varepsilon + \frac{F(u^\varepsilon) - v^\varepsilon}{\varepsilon},$$

with nonmonotone reaction function $F$. As speed of reaction tends to infinity ($\varepsilon \to 0$), the concentration of non-diffusing component $u^\varepsilon$ exhibits fast oscillations. We identify precisely its Young measure which, as a by-product, proves strong convergence of the diffusing component $v^\varepsilon$, a result that is not obvious at all from a priori estimates. Our work is based on analysis of regularization for forward-backward parabolic equations by Plotnikov [3]. We rewrite his ideas in terms of kinetic functions which clarifies the method, brings new insights, relaxes assumptions on model functions and provides a weak formulation for the evolution of the Young measure. Finally, in [2] we refine method of Plotnikov by application of classical Radon-Nikodym theorem.

This is a joint work with Benoît Perthame (Sorbonne University, Paris).

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# Dark Matter Models

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**Abstract**

The authors of ♐ have lately proposed an alternative dark matter model to non-rotating black hole suggestion of Sagittarius A* in the Galactic Centre, see also references therein. We analyzed some mathematical aspects of the suggested dark matter models recently in ↬.

♐ EA Becerra-Vergara, CR Argüelles, A Krut, JA Rueda, R Ruffini *Hinting a dark matter nature of Sgr A* via the S-stars* Monthly Notices of the Royal Astronomical Society: Letters **505** (2021) 64–68 doi.org/10.1093/mnrasl/slab051

↬ D Bors, R Stańczy *Models of particles of the Michie-King type* Communications in Mathematical Physics **382** (2021) 1243–1262 doi.org/10.1007/s00220-021-03981-8

# Discrete De Giorgi estimates and the finite element approximation of chemically reacting biological fluids

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**Abstract**

The talk is concerned with the convergence analysis of finite element methods for the approximate solution of a system of nonlinear elliptic partial differential equations that arise in models of chemically reacting viscous incompressible biological fluids. The shear-stress appearing in the model involves a power-law type nonlinearity, where, instead of being a fixed constant, the power law-exponent is a function of a spatially varying nonnegative concentration function, which, in turn, solves a nonlinear convection-diffusion equation. In order to prove the convergence of the sequence of finite element approximations to a solution of this coupled system of nonlinear PDEs, a uniform Hölder norm bound needs to be derived for the sequence of finite element approximations to the concentration in a setting, where the diffusion coefficient in the convection-diffusion equation satisfied by the concentration is merely an $L^\infty$ function. This necessitates the development of a finite element counterpart of the De Giorgi–Nash–Moser theory. Motivated by an early paper by Aguilera and Caffarelli (1986) in the simpler setting of Laplace's equation, we derive such uniform Hölder norm bounds on the sequence of continuous piecewise linear finite element approximations to the concentration. We then use results from the theory of variable-exponent Sobolev spaces equipped with a Luxembourg norm, Minty's method for monotone operators and an extension to variable-exponent Sobolev spaces of the finite element version of the Acerbi–Fusco Lipschitz-truncation method, originally developed in classical Sobolev spaces in collaboration with Lars Diening and Christian Kreuzer (SIAM J. Numer. Anal. 51(2): 984–1015 (2014)), to pass to the limit in the coupled system of nonlinear PDEs under consideration.

The talk is based on joint work with Seungchan Ko and Petra Pustějovská, and recent results obtained in collaboration with Lars Diening and Toni Scharle.

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# Weak solutions for the Stokes system for compressible fluids with general pressure

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**Abstract**

I will talk about existence and uniqueness of global in time weak solutions for the Stokes system for compressible fluids with a general, non-monotone pressure. We construct the solution at the level of Lagrangian formulation and then define the transformation to the original Eulerian coordinates. For nonnegative and bounded initial density the solution is also nonnegative for all $t$ and belongs to $L^\infty([0,\infty) \times \mathbb{T}^d)$. A key point of our considerations is the uniqueness of such transformation. Since the velocity might not be Lipschitz continuous, we develop a method which relies on the results of Crippa & De Lellis, concerning regular Lagriangian flows. The uniqueness is obtained thanks to the application of a certain weighted flow and detail analysis based on the properties of the $BMO$ space.

# Euler-Poisson equation - weak-strong uniqueness principle for dissipative measure-valued solutions

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**Abstract**

We will start with the statement of weak-strong uniqueness principle for general hyperbolic conservation laws and show that Euler-Poisson fails to fit into this framework. We consider several pressureless variants of the compressible Euler equation driven by nonlocal repulsion-attraction and alignment forces with Poisson interaction. Under an energy admissibility criterion, we prove existence of global *measure-valued solutions*, i.e., very weak solutions described by a classical Young measure together with appropriate concentration defects. We then investigate the evolution of a relative energy functional to compare a measure-valued solution to a regular solution emanating from the same initial datum. This leads to a (partial) weak-strong uniqueness principle.

# The modelling of a capillary rise dynamics using a nonlinear differential equation

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**Abstract**

Capillary rise is an extraordinary physical phenomenon that is ubiquitous in nature and the effects of which we meet in our everyday life. Mainly in this way, water and various nutrients are replenished in plants. Capillarity play also an important role in industry. The governing equation describing the change in the height of the liquid during the capillary flow in thin tube is a nonlinear ordinary differential equation of the second order. Due the fact that we only consider the flow in one direction (vertical tube) the dynamics of water height inside the tube can be successfully expressed in terms of ordinary differential equation. In my talk, I am going to discuss the classical mathematical model that describes the considered physical phenomenon, together with the presentation of some of its improvements.

Furthermore, during my talk I would like to present the most important results concerning the governing equations that models the phenomenon of capillary rise in a vertical thin tube [1, 2, 3]. In particular, I would like to present the theorem that asserts the existence and uniqueness of the solution of the discussed nonlinear differential equations. Next, I am going to present the results concerning the problem of changing the character of the capillary rise when the value of the dimensionless parameter present in the equation crosses a certain critical value. To complete the analysis of the dependence of the exact solution on the dimensionless parameter, the singular perturbation analysis for the limiting value of the mentioned parameter will be given.

The description of the nonlinear oscillation that occurs for some values of the nondimensional parameter may be a nontrivial example of the application of standard theory to nonlinear problems. A proper analysis of the solution of a nonlinear equation requires the use of nonstandard methods. Finally, the results presented during the talk may be helpful to better understand the features of the capillary flow and may be useful in the study of differential equations with the similar nonlinear component.

## References

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[2] Ł. Płociniczak and M. Świtała. Asymptotic behaviour of a solution to a nonlinear equation modelling capillary rise. *Physica D: Nonlinear Phenomena*, 406:132394, 2020.

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# Numerical solution of the Schrödinger equation on the whole real line

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**Abstract**

A common approach to approximating the solution to the Schrödinger equation is Fourier spectral methods, because special properties of the Fourier basis allow fast algorithms to compute them. However, this forces the solution to live in the periodic domain of the Fourier basis, which conflicts with the Schrödinger equation's natural setting in Euclidean space. This actually poses a time limit on the accuracy of the approximation, because any waves which encounter the edge of the periodic domain experience a non-physical wrap-around effect. In this talk, we discuss whether the success of Fourier spectral methods can be translated to the whole real line.

This is joint work with Katharina Schratz (Sorbonne), Arieh Iserles (Cambridge), Karen Luong (Cambridge), and Karolina Kropielnicka (IM PAN).

# The challenge of non-local operators and mixed dimension in simulation and modelling

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**Abstract**

In this talk, we address some mathematical and numerical challenges of non-linear or non-local partial differential equations in applications. We use a variety of applications ranging from porous-media flow systems, fluid and structural mechanics to biomedical engineering. The special challenges of variational inequalities, mixed-dimensions and non-integer differential operators are illustrated. We show the flexibility of abstract mathematical concepts and discuss limitations in theory and convergence.

# Singular limit for a system without relative entropy

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**Abstract**

I will discuss the low Mach and low Froude numbers limit for the compressible Navier-Stokes equations with degenerate, density-dependent, viscosity coefficient, in the strong stratification regime. The talk is based on a joint paper with Francesco Fanelli from Univ. Lyon. Our main result is the proof of convergence to the generalised anelastic approximation, which is used extensively to model atmospheric flows. We considered the case of a general pressure law with singular component close to vacuum, general ill-prepared initial data, and periodic boundary conditions.

# Shadowing of nontransversal heteroclinic chains in cubic defocusing NLS on the plane

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**Abstract**

In the present paper we deal with the problem of shadowing a nontransversal chain of heteroclinic connections between invariant sets (fixed points, periodic orbits, etc). The motivation for us is the work [CK+] (see also [GK]) on the transfer of energy to high frequencies in the nonlinear Schrodinger equation (just NLS from now on). From the dynamical systems viewpoint there is one remarkable feature of the construction in [CK], namely that the authors were able to shadow a non-transversal highly degenerated chain of heteroclinic connections between some periodic orbits. The length of the chain is arbitrary, but finite. Neither in [CK] nor in [GK] we were able to find a clear geometric picture showing how this is achieved, so it could be easily applicable to other systems. In this work we present a mechanism, which we believe gives a geometric explanation of what is happening. Moreover, we strive to establish an abstract framework, which will make it easier to apply this technique to other systems, both PDEs and ODEs, in questions related to the existence of diffusing orbits. The term *diffusing orbit* relates to the Arnold's diffusion for the perturbation of integrable Hamiltonian systems. We will call diffusing orbit an orbit shadowing a chain of heteroclinic connections, and occasionally the existence of such an orbit will be referred to as the diffusion.

In our picture we think of evolving a disk of dimension $k$ along a heteroclinic transition chain and when a given transition is not transversal, then we 'drop' one or more dimensions of our disk, i.e., we select a subdisk of lower dimension "parallel to expanding directions in future transitions". After at most $k$ transitions, our disk is a single point and we cannot continue further. We will refer to this phenomenon as the *dropping dimensions* mechanism. Since this is a new mechanism, we have found it convenient to include several figures to illustrate the main differences between transversal and non-transversal heteroclinic chains. While thinking about disks has some geometric appeal, we consider instead in our construction a thickened disk called h-set and our approach is purely topological (just as the one presented in [CK+]).

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