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Derivation of macroscopic equations for individual cell-based models.

Typically, in individual cell-based models cells interact by means of some pair potential and are assumed to evolve according to some stochastic or deterministic dynamics. Because these models try to describe interaction between individuals they are often called microscopic models. They can describe quite complicated phenomena. The rule which governs the cells dynamics can be usually easily implemented and the numerical simulation might give some solutions, in particular in the case of cellular automata models. On the other hand, if we try to give a precise mathematical description it is usually complicated and the mathematical analysis of such models is very difficult if possible. Often it is also very difficult to identify the most relevant parameters or group of parameters and its influence on the dynamics.

Our talk will be focused on a very particular type of models that are analogous to many of the model studied in the literature. We will assume that the centres of the cells evolve according to ordinary differential equation

$$\frac{d}{dt}X_N(k,t) = -\sum_{\substack{i=1\\ i\neq k}}^{N} \nabla V_N(X_N(k,t) - X_N(i,t))\,,$$

where N is a number of cells and functions $X_N(k,t)$ describe the position of the k-th cell. We assume that dominant effect in the dynamics is cell friction and for that reason only one derivative appears on the left-hand side. We will derive a equation that can describe a macroscopic behaviour of the system. In the case of "long-range" potentials, this is when one cell/particle interacts with many others the evolution of the cell/particle density is described by a type of porous-medium equation. On the other hand, if interaction are "short", this is a support of potential V is of the order of typical distance between cells/particles the structure of the equilibrium state of the microscopic system appears in the macroscopic equation. In 1-D this leads to a version of porous-medium equation discrete in space. However for higher dimensions a directional densities have to be considered.

References

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