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Transient chaos measurements using finite-time Lyapunov exponents in model of population dynamics

The family of logistic maps is the best known nonlinear model of population dynamics. The typical analysis of this model is concentrated on its asymptotic behaviour. Special attention is payed to properties of trajectories generated by the maps inside periodic windows, where the periodic behaviour occurs [1]-[3]. However such periodic behaviour is preceded by chaotic transient behaviour. The duration of such transient chaos can be prolonged [4],[5] .

We propose a model for estimating the duration of transient chaos based on calculation of finite-time Lyapunov exponents. Lyapunov exponents belong to the most useful tools applied for measuring sensitivity to initial conditions in the case of asymptotic chaos. We used Lyapunov exponents for characterizing sensitivity to initial conditions in the case of transient chaos. Before doing that we modify the notion of finite-time Lyapunov exponent averaging them over a set of initial conditions and we report results of tests providing evidence in favor of correctness of such an approach. We also present a model reproducing correctly variation in time of the finite-time Lyapunov exponents corresponding to transient chaos.The dependence on time is verified by comparing theoretically predicted values with those obtained numerically.

References

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