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New exact solutions of mathematical models describing peritoneal transport

Mathematical description of fluid and solute transport between blood and dialysis fluid in the peritoneal cavity has not been formulated fully yet, in spite of the well known basic physical laws for such transport. Recent mathematical, theoretical and numerical studies introduced new concepts on peritoneal transport and yielded better results for the transport of fluid and osmotic agent [1]–[4]. However, the problem of a combined description of osmotic ultrafiltration to the peritoneal cavity, absorption of osmotic agent from the peritoneal cavity and leak of macromolecules (proteins, e.g., albumin) from blood to the peritoneal cavity has not been addressed yet. Therefore, we present here a new extended model for these phenomena and investigate its mathematical structure. The model is based on a three-component nonlinear system of two-dimensional partial differential equations with the relevant boundary and initial conditions. In the particular case, this model produces one, which was studied earlier in papers [1]–[3]. The non-constant steady-state solutions of the model obtained are studied. The realistic restrictions on the parameters arising in the model were established with the aim to obtain exact formulae for the non-constant steady-state solutions. As result, the exact formulae for the density of fluid flux from blood to tissue and the volumetric flux across the tissue were constructed, and two linear autonomous ordinary differential equations to find the glucose and albumin concentrations were derived. The analytical results were checked, whether they are applicable for the description of the glucose-albumin transport in peritoneal dialysis.

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