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The stochastic Morris-Lecar neuron model embeds a one-dimensional diffusion and its first-passage-time crossings

Stochastic leaky integrate-and-fire models, i.e. one-dimensional mean-reverting diffusions, are popular tools to describe the stochastic fluctuations in the neuronal membrane potential dynamics due to their simplicity and statistical tractability. They have been widely applied to gain understanding of the underlying mechanisms for spike timing in neurons, and have served as building blocks for more elaborate models. Especially the Ornstein-Uhlenbeck process is popular, but also other models like the square-root model or models with a non-linear drift are sometimes applied. However, experimental data show varying time constants, state dependent noise, a graded firing threshold and time-inhomogeneous input, and higher dimensional, more biophysical models are called for.

The stochastic Morris-Lecar neuron is a two-dimensional diffusion which includes ion channel dynamics. We show that in a neighborhood of its stable point, it can be approximated by a two-dimensional Ornstein-Uhlenbeck modulation of a constant circular motion. The associated radial Ornstein-Uhlenbeck process is an example of a leaky integrate-and-fire model prior to firing. A new model constructed from a radial Ornstein-Uhlenbeck process together with a simple firing mechanism based on detailed Morris-Lecar firing statistics reproduces the interspike interval distribution, and has the computational advantages of a one-dimensional model. The result justifies the large amount of attention paid to the leaky integrate-and-fire models.