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Existence of Positive Almost Periodic or Ergodic Solutions for Some Neutral Nonlinear Integral Equations

As we all know, the existence of periodic solutions of functional differential equations (FDE) has great theoretical and practical significance and is one of the problems of great interest to scholars in the field. Since Yoshizawa [2] presented an excellent result for the existence of periodic solutions to FDE with bounded delay, Cooke and Huang [3], Burton and Hatvani [1] generalized Yoshizawa's result to FDE with infinite delay. We remark that, in the nature, there is no phenomenon which is purely periodic, this gives the idea to consider the almost periodic situation.

In this paper, we consider the following neutral nonlinear integral equation

$$x(t) = \gamma x(t-\sigma) + (1-\gamma)\int_{t-\sigma}^{t} f(s, x(s))\, ds, \tag{1}$$

where $0 \leq \gamma < 1$, $\sigma > 0$ and $f : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}^+$ is a continuous map.

We give sufficient conditions which guarantee the existence of almost periodic solutions for Equation (1). We also treat the ergodic solutions that means the asymptotically almost periodic, the weakly almost periodic and pseudo almost periodic solutions. Hypotheses of our results do not impose that the function $f(t, .)$ is monotone. To state our results, we use a variant of Hilbert's projective metric on a subset of a space of continuous and bounded functions.

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