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Buffered calcium waves with mechano-chemical effects

We analyze the following system of equations:

$$(1)\qquad \begin{aligned} \frac{\partial c}{\partial t} &= D\frac{\partial^2}{\partial x^2}c + g(c) + \sum_{i=1}^{n} G_i(c, v_i) \; + R(c, \theta, J_1, J_2) \\ \frac{\partial v_i}{\partial t} &= D_i \frac{\partial^2}{\partial x^2} v_i - G_i(c, v_i), \quad i = 1, \dots, n, \end{aligned}$$

$$(2)\qquad 0 = \nabla \cdot \left\{ \frac{E}{1+\nu}\left[\varepsilon + \frac{\nu}{1-2\nu}\theta \mathbf{I}\right] + \mu_1 \frac{\partial \varepsilon}{\partial t} + \mu_2 \frac{\partial \theta}{\partial t}\mathbf{I} + \tau(c)\mathbf{I} \right\} - \vartheta \mathbf{u}.$$

where c denotes the concentration of free cytosolic calcium ions, v_i the concentration of the i-th buffer, ε the strain tensor, $\mathbf{u}$ displacement field, τ active concentration stress resulting from the actomyosin traction $\tau(c)$. We assume that the ratio $(\mu_1 + \mu_2)/E$ is sufficiently small. We prove the existence of travelling waves to the above system, analyze the influence of viscosity on the speed of the wave and give the explicit formulae for some specific solutions. We confine ourselves to three geometrical cases: bulk medium (large in every direction), infinite plane layer of sufficiently small width and long cylinder of sufficiently small radius.

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