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Global stability analysis with a discretization approach for an age-structured SIR epidemic model

The global stability analysis for each equilibrium of an age-structured SIR epidemic model is carried out. After discretizing the model that is a system of PDE with respect to the age variable, we obtain a multigroup epidemic model that is a system of ODE and can apply the classical method of Lyapunov, a recently developed graph-theoretic approach and a monotone iterative method in order to show the global asymptotic stability of the disease-free equilibrium for $R_0 \leq 1$, and the global attractivity of an endemic equilibrium for $R_0 > 1$, where R_0 is the basic reproduction number. Although for the original PDE model the possibility of local instability of an endemic equilibrium was shown even for $R_0 > 1$, for the discretized version of it we can obtain the aforementioned global attractivity result, and this implies that the possibility of periodic solutions might be ruled out from the model, which has been discussed as an open question for more than two decades. Numerical simulation provides an example indicating that the numerical solutions of the two PDE and ODE systems become closer to each other as the step size of discretization decreases.

References

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