Víctor M. Pérez-García
Departamento de Matemáticas, E.T.S. de Ingenieros Industriales & IMACI-Instituto de Matemática Aplicada a la Ciencia y la Ingeniería, Universidad de Castilla-La Mancha, 13071, Ciudad Real, Spain
e-mail: `victor.perezgarcia@uclm.es`
J. Belmonte-Beitia, G. F. Calvo, D. Diego
Departamento de Matemáticas, IMACI-Instituto de Matemática Aplicada a la Ciencia y la Ingeniería, Universidad de Castilla-La Mancha, 13071, Ciudad Real, Spain

Bright solitons in malignant gliomas

Malignant gliomas are the most common and deadly brain tumors. Survival for patients with glioblastoma multiforme (GBM), the most aggressive glioma, although individually variable, is in the range of 10 months to 14 months after diagnosis, using standard treatments which include surgery, radiotherapy, chemotherapy (temozolamide and antiangiogenic drugs such as bevacizumab) [1]. GBM is a rapidly evolving astrocytoma that is distinguished pathologically from lower grade gliomas by the presence of necrosis and microvascular hyperplasia.

Many mathematical models have been proposed to describe specific aspects of GBM cell lines in vitro [2,3] and the tumor growth in vivo even under the action of radiotherapy [4-6]. Recently some applications of these models have been used to predict the survival of patients after surgical resection of GBMs [7].

Most of the mathematical models in use for GBM are based on a simple reaction-diffusion equation: the Fischer equation [8]. This equation in one spatial dimensions has travelling wave solutions of kink type but has no travelling wave solutions in higher dimensions [9].

In this communication we will first describe two extensions of the Fischer equation, the first one accounting for the necrotic core and the normal tissue and the second one incorporating the vasculature. We will then show how bright tumor solitons arise spontaneously separating a kink of normal tissue from a kink of growing necrotic tissue. We will relate the soliton parameters (corresponding to the active tumor area) to the clinically relevant parameters. The effect of surgical resection on the nonlinear dynamics of the system will be discussed. In our analysis we will resort to different tools of the theory of nonlinear waves: time-dependent variational methods [10], moment methods [11], Lie group theory methods [12], similarity transformations [13], and numerical simulations. We will also discuss the existence of multidimensional travelling waves employing analytical methods and advanced numerical methods incorporating the system's geometry [14].

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