Piotr Podziemski
Faculty of Physics, Warsaw University of Technology
e-mail: podziemski@if.pw.edu.pl
Jan J. Żebrowski
Faculty of Physics, Warsaw University of Technology
e-mail: zebra@if.pw.edu.pl

Modeling of the human atrium using Liénard equations

Liénard systems can be used for modeling oscillatory behaviour of many phenomena - starting from chemical reactions, through neuron excitability [1], up to the action potential in the heart muscle. The universality of the Liénard systems and the rather well-established mathematical knowledge about them creates a flexible framework for designing simple models. Such models are very robust and computationally efficient. On the contrary, the existing physiological ionic channel models of cardiac cells are too complex to allow an investigation of long time dynamical properties of the heart. As a consequence, very rarely do they address the problem of heart rate variability comparable with portable ECG recordings.

We focus on the simulation of human atria, where the dynamics of action potential propagation affects the sinus rythm the most. In the model of the right atrium proposed here, we describe the various anatomical parts of the atrium by means of different equations but all of the same class of Liénard equations. The two nodes - the sinoatrial and the atrioventricular node are modeled by diffusively coupled modified van der Pol-Duffing oscillators while the atrial muscle tissue is currently represented by a diffusively coupled modified FitzHugh-Nagumo system.

Models of the sinoatrial and atrio-ventricular nodes were developed taking into account physiologically important properties such as the phase response curve, the refraction period and threshold potential. Several modifications of the models presented in [2] allowed to achieve a more physiological behaviour of the model. The effect of the autonomous nervous system activity is incorporated into the model in a simple way.

We performed a series of simulations of the atrium, with differing anatomical simplifications varying from a simple 1 dimensional chain of oscillators to a two-dimensional mapping of the atrium with chosen anatomical details included. The simulations allowed to reconstruct such effects as the AV node reentry tachycardia - both in an extended one dimensional model and in the 2D simulation, the phase relations between sinus rhythm and the location and properties of an ectopic source and their effects on the resultant rhythm.

References

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[2] J.J. Żebrowski, P. Kuklik, T. Buchner. R. Baranowski, *Assessment and clinical applications of cardiovascular oscillations* IEEE Eng. In Med. And Biol. Mag., Nov./Dec. 2009 .