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Asymptotic properties of stochastic symbiosis model

We discuss the influence of various stochastic perturbations on symbiosis system. We consider the following system of stochastic equations

$$(1)\begin{cases} dX(t) = ((a_1 + b_1Y(t) - c_1X(t))\,dt + \rho_{11}\,dW_1(t) + \rho_{12}\,dW_2(t))\,X(t) \\ dY(t) = ((a_2 + b_2X(t) - c_2Y(t))\,dt + \rho_{21}\,dW_1(t) + \rho_{22}\,dW_2(t))\,Y(t), \end{cases}$$

which describes relations between two populations living in symbiosis. We assume that $a_i, b_i, c_i > 0$ $(i = 1, 2)$ are positive constants, $W_1(t)$, $W_2(t)$ are two independent standard Wiener processes, $X(t)$, $Y(t)$ are stochastic processes which represent, respectively, the first and the second population. We consider three kinds of stochastic perturbations:

(i) weakly correlated, i.e. $\rho_{11}\rho_{22} - \rho_{12}\rho_{21} \neq 0$;
(ii) strongly correlated, i.e. $\rho_{11} > 0$, $\rho_{21} > 0$, $\rho_{12} = 0$, $\rho_{22} = 0$;
(iii) only one population is stochastically perturbed, by symmetry we assume that the second population is perturbed, i.e. $\rho_{11} = 0$, $\rho_{21} > 0$, $\rho_{12} = 0$, $\rho_{22} = 0$.

First we show the existence, uniqueness, positivity and non-extinction property of the solutions of system (1) on the assumption that $b_1b_2 < c_1c_2$. Next we prove that the probability distributions of the process $(X(t), Y(t))$ are absolutely continuous with respect to the Lebesgue measure. Let $U(x, y, t)$ be the density of the distribution of $(X(t), Y(t))$. We give a sufficient and a necessary condition for asymptotic stability of system (1), i.e. the convergence of $U(x, y, t)$ to an invariant density $U_*(x, y)$. In the case when this system is not asymptotically stable, we prove that $\lim_{t\to\infty} Y(t) = 0$ a.e. We also show that in this case $\lim_{t\to\infty} X(t) = 0$ a.e. or the probability distributions of the process $X(t)$ converge weakly to some probability measure. We give a biological interpretation of these results.

References

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[2] U. Skwara, *A stochastic model of symbiosis with degenerate difussion process* Ann. Polon. Math. **98.2** 111–128.