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## Predicting the period in seasonally driven epidemics

Seasonality strongly affects the transmission and spatio-temporal dynamics of many infectious diseases, and is often an important cause for their recurrence. However, there are many open questions regarding the intricate relationship between seasonality and the complex dynamics of infectious diseases it gives rise to. For example, in the analysis of long-term time-series of childhood diseases, it is not clear why there are transitions from regimes with regular annual dynamics, to regimes in which epidemics occur every two or more years, and vice-versa. The classical seasonally-forced SIR epidemic model gives insights into this phenomena but due to its intrinsic nonlinearity and complex dynamics, the model is rarely amenable to detailed mathematical analysis. Making sensible approximations we analytically study the threshold (bifurcation) point of the forced SIR model where there is a switch from annual to biennial epidemics. We derive, for the first time, a simple equation that predicts the relationship between key epidemiological parameters near the bifurcation point. The relationship makes clear that the epidemic period will decrease if either the birth-rate ( $\lambda$ ) or basic reproductive ratio ( $R_0$ ) is increased sufficiently, or if the strength of seasonality ( $\beta$ ) is reduced sufficiently. These effects are confirmed in simulation studies and are also in accord with empirical observations. For example, in the pre-vaccination era, the increase in birth-rate in the United States and in the United Kingdom was the factor responsible for driving measles dynamics from biennial to annual oscillations. Moreover, it is argued that the strong seasonality in India (high  $\beta$ ) may be responsible for the erratic polio outbreaks. Correspondingly, our equation identifies the first bifurcation in the expected period-doubling route to chaos that continues as seasonality increases.