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Origins of Allometric Growth: A Contemporary Perspective

The theoretical allometry relation (AR) between the size of an organism Y and that of an organ within the organism X is of the form $X = aY^b$ and has been known for nearly two centuries. The allometry coefficient a and allometry exponent b have been fit by various data sets over that time. In the last century the phenomenological field of allometry has found its way into almost every scientific discipline and the ARs have been reinterpreted with Y still being the size of a host network and X a function of the network. For example, in biology the measure of size is often taken to be the total body mass and the function is the metabolic rate, or heart rate, breathing rate, or longevity of animals. Most theories purporting to explain the origin of ARs focus on establishing the proper value of b entailed by reductionist models, whereas a few others use statistical arguments to emphasize the importance of a.

On the other hand, statistical data analysis indicates that empirical ARs are obtained with the replacements $X \to \langle X \rangle$ and $Y \to \langle Y \rangle$ and the brackets denote an average over an ensemble of realizations of the network and its function. Networks in which these empirical ARs are established include the metabolism of animals, the growth of plants, species abundance in econetworks, the geomorphology of rivers, and many more. The resulting empirical AR can only be derived from the theoretical one by averaging under conditions that are incompatible with real data. Consequently another strategy for finding the origin of ARs is required and for this we turn to the probability calculus and fractional derivatives.

We assume that the statistics of living networks can be described by fractional diffusion equations (FDEs) and hypothesize that FDEs can explain the origin of ARs. We obtain the Fourier-Laplace transform of the general solution to the FDE that contains both historical information and nonlocal influences on the dynamic variables, that is, fractional derivatives in both time and phase space, complexity commonly found in living networks. The scaling properties of the resulting solution to the FDE enable us to interrelate the network's size and function by means of the mechanism of strong anticipation. The analysis shows that strong anticipation and scaling taken together support the hypothesis and is sufficient to explain the origin of empirical ARs.