On geometric complexity of Julia sets

Będlewo, 18-23 March 2018

Abstracts of talks

Accesses to infinity from Fatou components

Krzysztof Barański

(Joint work with Núria Fagella, Xavier Jarque and Bogusława Karpińska)

We study the boundary behaviour of a meromorphic map f on its simply connected invariant Fatou component U. To this aim, we develop the theory of accesses to boundary points of U and their relation to the dynamics of f. In particular, we establish a correspondence between invariant accesses from U to infinity or weakly repelling fixed points of f and boundary fixed points of the associated inner function on the unit disc. We apply our results to describe the accesses to infinity from invariant Fatou components of the Newton maps.

A landing theorem for entire transcendental functions with bounded postsingular set

Anna Miriam Benini

(Joint work with Lasse Rempe-Gillen)

The *Douady-Hubbard landing theorem* for periodic external rays is one of the cornerstones of the study of polynomial dynamics. It states that, for a complex polynomial f with bounded postcritical set, every periodic external ray lands at a repelling or parabolic periodic point, and conversely every repelling or parabolic point is the landing point of at least one periodic external ray. We prove an analogue of this theorem for an entire function f with bounded postsingular set for which the escaping set consists of curves, called dynamic rays: every periodic ray lands at a repelling or parabolic periodic point, and conversely every repelling or periodic parabolic point is the landing point of at least one periodic ray. More generally, a similar proof extends to show that every point of a hyperbolic set K of f is the landing point of a ray.

Julia sets with a wandering branching point
Jordi Canela

According to the Thurston No Wandering Triangle Theorem, a branching point in a locally connected quadratic Julia set is either preperiodic or precritical. Blokh and Oversteegen proved that this result does not extend to higher degree polynomials using laminations. In this talk we approach the existence of wandering non-precritical branching points for cubic polynomials from the point of view of perturbations of postcritically finite maps. We will present an iterative method which starts with an 'admissible' postcritically finite cubic polynomial and converges to a map with wandering non-precritical branching points.

Dynamics of rescaled limits of quadratic polynomials
Dzmitry Dudko
(Joint work with Mikhail Lyubich)

Consider the unstable manifold of a pacman renormalization operator. Maps on the unstable manifold are rescaled limits of quadratic polynomials. Every such limit admits a maximal extension to a σ-proper branched covering of the complex plane. Using methods and ideas from transcendental dynamics, we show that certain maps on the unstable manifold are hybrid equivalent to quadratic polynomials. This allows us to construct a stable lamination in the space of pacmen. Combined with hyperbolicity of pacman renormalization, we obtain various scaling results near the main cardioid of the Mandelbrot set. As a consequence, the Mandelbrot set is locally connected at certain infinitely renormalizable parameters of bounded satellite type.

Non-escaping endpoints of Cantor bouquet Julia sets
Vasiliki Evdoridou

There are large classes of transcendental entire functions whose Julia set consists of an uncountable union of disjoint curves each of which joins a finite endpoint to infinity (a Cantor bouquet). Mayer was the first to study the topology of the set of endpoints of these curves for some functions in the exponential family. Recenlty, Alhabib and Rempe-Gillen extended his result and also proved analogous results for the escaping endpoints of large classes of entire functions. Starting from a famous function studied by Fatou and the exponential family, we obtain results on the topology of the set of non-escaping endpoints of these curves. In particular, we show that for some classes of functions whose Julia set is a Cantor bouquet the union of non-escaping endpoints with infinity is a totally separated set. This result gives a strong dichotomy between the topological properties of escaping and non-escaping endpoints for the functions we consider.

The support of the bifurcation measure has positive volume

Thomas Gauthier

(Joint work with Matthieu Astorg, Nicolae Mihalache and Gabriel Vigny)

The moduli space $\mathcal{M}_d$ of degree $d \geq 2$ rational maps can naturally be endowed with a measure μ_{bif} detecting maximal bifurcations, called the *bifurcation measure*. We prove that the support of the bifurcation measure μ_{bif} has positive Lebesgue measure. To do so, we establish a general criterion for the conjugacy class of a rational map to belong to the support of μ_{bif} and we exhibit a "large" set of Collet-Eckmann rational maps which satisfy that criterion. As a consequence, we get a set of Collet-Eckmann rational maps of positive Lebesgue measure which are approximated by hyperbolic rational maps.

Inaccessibility of hyperbolic components for anti-holomorphic dynamics

Hiroyuki Inou

(Joint work with Sabyasachi Mukherjee)

In the anti-holomorphic unicritical family $\bar{z}^d + c$, objects accumulating to a hyperbolic component of odd period are often "wiggly" and converge to an arc of positive length. This suggests that if decorations converging to such a hyperbolic component are sufficiently complicated, then the hyperbolic component is not accessible from the escape locus. We explain that the inaccessibility is related to some property of Julia sets, and then we would discuss how one can prove such a property.

On the derivative of the Hausdorff dimension of the quadratic Julia sets

Ludwik Jaksztas

Let $d(c)$ denote the Hausdorff dimension of the Julia set $J(z^2 + c)$. We will investigate the derivative $d'(c)$, for real c converging to a parabolic parameter c_0. First, we will prove that $d'(c)$ tends to infinity, when $c \nearrow 1/4$. Next, we will see that $d'(c)$ tends to a constant or minus infinity depending on the value $d(c_0)$, where c_0 is a parabolic parameter with two petals.

Conformal measures for meromorphic maps

Bogusława Karpińska

In this talk we discuss the relation between the existence of a conformal measure on the Julia set $J(f)$ of a transcendental meromorphic map f and the existence of a zero of the topological pressure function with respect to the spherical metric. The talk is based on a joint work with Krzysztof Barański and Anna Zdunik.

Dynamical zeta functions and prime orbit theorems in complex dynamics

Zhiqiang Li

(Joint work with T. Zheng)

Analogues of the Riemann zeta function were first introduced into geometry by A. Selberg and into dynamics by M. Artin, B. Mazur, and S. Smale. Analytic studies of such dynamical zeta functions yield quantitative information on the distribution of closed geodesics and periodic orbits.

We obtain the first Prime Orbit Theorem, as an analogue of the Prime Number Theorem, in complex dynamics outside of hyperbolic maps, for a class of branched covering maps on the 2-sphere called expanding Thurston maps f. More precisely, we show that the number of primitive periodic orbits of f, ordered by a weight on each point induced by a non-constant real-valued Hölder continuous function on S^2 satisfying some additional regularity conditions, is asymptotically the same as the well-known logarithmic integral, with an exponentially small error term. Such a result follows from our quantitative study of the holomorphic extension properties of the associated dynamical zeta functions and dynamical Dirichlet series.

In particular, the above result applies to postcritically-finite rational maps whose Julia set is the whole Riemann sphere. Moreover, we prove that the regularity conditions needed here are generic; and for a Lattès map f, a continuously differentiable function satisfies such a condition if and only if it is not cohomologous to a constant.

Pacman Renormalization and scaling of satellite Mandelbrot copies near Siegel points

Mikhail Lyubich

(Based upon joint work with Dima Dudko and Nikita Selinger)

In the 1980s Branner and Douady discovered a surgery relating various limbs of the Mandelbrot set. We put this surgery in the framework of "Pacman Renormalization Theory" that combines features of quadratic-like and Siegel renormalizations. Siegel renormalization periodic points (constructed by McMullen in the 1990s) can be promoted to pacman renormalization periodic points. We prove that these periodic points are hyperbolic with one-dimensional unstable manifold.

Dynamics of Schwarz reflections: mating polynomials with groups

Sabyasachi Mukherjee

(Joint work with Mikhail Lyubich and Nikolai Makarov)

We consider a family of anti-holomorphic dynamical systems generated by Schwarz reflections with respect to the cardioid and a family of circumscribed circles. We show that every post-critically finite map in this family arises as the mating of a post-critically finite quadratic anti-holomorphic polynomial and the ideal triangle group. We also describe a combinatorial model for the "connectedness locus" of this family.

Complex Feigenbaum Phenomena of High Type

Mohammad Pedramfar

(Joint work with Davoud Cheraghi)

In this talk we analyse the dynamics of a class of infinitely satellite renormalizable maps with high type combinatorics, addressing infinite renormalisation structures with degenerating geometries. We establish a topological model for the post-critical sets of these maps, leading to a dichotomy identified by an arithmetic condition in the spirit of the Herman-Yoccoz condition for linearisation of analytic circle diffeomorphisms. That is, the post-critical set is either a Cantor set of points, or a *hairy Cantor set*, a topological object enjoying similar topological features as the Cantor set.
On the geometric aspect, among other results, we find an optimal arithmetic condition that implies the post-critical set is a Cantor set with bounded geometry.

Jakobson's Theorem via Yoccoz puzzles

Mitsuhiro Shishikura

Real quadratic maps $p_c : x \mapsto x^2 + c$ (or $f_a : x \mapsto ax(1-x)$) on $\mathbb{R}$ have a wide spectrum of dynamical behaviors, from those having attracting periodic orbits ("regular dynamics") to those with an absolute continuous invariant measure with respect to the Lebesgue measure ("stochastic dynamics"). Jakobson's theorem states that there exists a parameter set of positive Lebesgue measure for which the dynamics is stochastic. This theorem was reproved and improved by Rychlik, Guckenheimer, Benedicks-Carleson, Tsujii. Furthermore Lyubich has obtained a theorem which says that for almost all parameters, the dynamics is either regular or chaotic. In this talk, we will discuss the classic Jakobson's theorem and try to prove it through Yoccoz puzzles. Yoccoz puzzles and parapuzzles are used in many aspects in the complex dynamics, including the above result by Lyubich. In our approach, once we set up the puzzles and parapuzzles, all the arguments reduce to the combinatorics about the recurrence of critical points, together with complex analytic theorem about annuli and area (modulus-area inequality and the insulation principle).

On simply-connected domains with connected preimage

David Sixsmith

We consider the following straightforward question: does there exist a transcendental entire function f, and two disjoint simply-connected domains U and V, such that $f^{-1}(U)$ and $f^{-1}(V)$ are both connected?

The answer is surprising, and contrary to a series of published proofs, dating back to 1970. As well as answering this question, we discuss the error in these proofs, and the dynamical questions that are reopened.

On the Lebesgue Measure of the Feigenbaum Julia set

Scott Sutherland

In joint work with Artem Dudko *(IMPAN)*, we show that the Julia set of the quadratic Feigenbaum map has Hausdorff dimension less than two and consequently zero Lebesgue measure, answering a long-standing open question. This is established by a combination of new estimation techniques and a rigorous computer-assisted computation.

Random conformal dynamical systems and random invariant measures

Anna Zdunik

In the context of random iterates of transcendental maps, random Julia and radial Julia sets will be defined. I will present recent results obtained in a collaboration with Mariusz Urbański. For a random iteration of non-hyperbolic exponential maps $z \mapsto \lambda \exp(z)$ (within some range of parameters) we prove the existence of random conformal and invariant measures supported on the radial Julia set, and describe the behoviour of a typical trajectory.