Dynamics, measures and dimensions

Będlewo, 7-12 April 2019

Abstracts of talks

Some remarks on mixing properties of infinite measure preserving transformations
Jon Aaronson

I'll review the Hopf-Krickeberg ratio mixing property with examples and discuss related ergodic properties.

Hausdorff measure of shrinking target sets
Balázs Bárány

We study the Hausdorff measure of the shrinking targets on self-conformal sets.

Let Λ be a finite set of symbols, and let $\Phi = \{f_i\}_{i\in\Lambda}$ be an iterated function system (IFS) of $C^{1+\varepsilon}$ conformal and contracting mappings such that the open set condition holds. Denote X the attractor of Φ, and for $\mathbf{i} = (i_1, \dots, i_n) \in \Lambda^*$ let $X_{\mathbf{i}} = f_{i_1} \circ \cdots \circ f_{i_n}(X)$. Moreover, let $\pi\colon \Lambda^{\mathbb{N}} \mapsto X$ be the natural projection, and let $\sigma\colon \Lambda^{\mathbb{N}} \mapsto \Lambda^{\mathbb{N}}$ be the left-shift operator. Let $\psi\colon \mathbb{N} \mapsto \mathbb{R}_+$ be a monotone decreasing function.

For any $x \in X$, denote $W(x, \psi)$ the shrinking target set,

$$\begin{aligned} W(x,\psi) &= \{y \in X : \|y - f_{\mathbf{i}}(x)\| \le \operatorname{diam}(X_{\mathbf{i}})\psi(|\mathbf{i}|) \text{ for inf. many } \mathbf{i} \in \Lambda^*\} \\ &= \pi\{\mathbf{i} \in \Lambda^{\mathbb{N}} : \|\pi(\sigma^n \mathbf{i}) - x\| \le \psi(n) \text{ for inf. many } n \ge 0\}. \end{aligned}$$

We will show that for every $s \ge 0$ and every open ball B

$$\mathcal{H}^s(W(x,\Psi) \cap B) = \begin{cases} 0 & \text{if } \sum_{\mathbf{i}\in\Lambda^*} (\operatorname{diam}(X_{\mathbf{i}})\psi(|\mathbf{i}|))^s < \infty \\ \mathcal{H}^s(X \cap B) & \text{if } \sum_{\mathbf{i}\in\Lambda^*} (\operatorname{diam}(X_{\mathbf{i}})\psi(|\mathbf{i}|))^s = \infty. \end{cases}$$

The Hausdorff dimension of the shrinking target sets was already studied by Hill and Velani, and later Baker studied the Hausdorff measure in the special case when $s = \dim_H X$. Altough one of the most common and most powerful tool to study the Hausdorff measure is the mass transference principle, introduced by Beresnevich and Velani, we will show that this technique cannot be applied in our setup in general.

This is a joint work with Demi Allen (Bristol).

Thermodynamic formalism for transcendental maps
Krzysztof Barański

We discuss some results obtained in the recent years establishing elements of the thermodynamic formalism for various classes of transcendental entire and meromorphic maps.

Lebesgue measure of Julia sets of entire functions
Walter Bergweiler

We study continuity properties of dynamical quantities while crosfamily of iterates fails to be normal. The escaping set is the set of points which tend to infinity under iteration. McMullen showed that these sets have positive measure for the sine and cosine function. Since then these results have been extended to various classes of functions. We give a new criterion implying that these sets have positive measure. For example, the results can be applied to Poincaré functions of certain polynomials.

Lyapunov exponents and bifurcations near Lattès maps
François Berteloot

Lattès maps are usually unstable and are actually a source of very strong bifurcations. For instance, a rigid Lattès map on the Riemann sphere is both accumulated by hyperbolic rational functions and by rational functions whose number of neutral periodic points is maximal. In any holomorphic family of endomorphisms of $\mathbb{P}^k$, the Hausdorff dimension of the bifurcation locus near an isolated Lattès map is maximal in any direction. We will explain how such results are related to the behaviour of Lyapunov exponents under perturbations.

Entropies of semigroup actions
Andrzej Biś

The talk is based on papers [1]-[4], where we consider entropy of a finitely generated semigroup G of continuous maps acting on a compact metric space X. In the first part of the talk, based on the joint paper with Mariusz Urbański [1], I will present a few basic properties of the topological entropy of G. The complexity of a semigroup can be desribed by several entropy-like quantities which are interrelated and reflect different features of semigroup dynamics ([2]). In general, for a finitely generated semigroup G there is no G-invarinat measure. Therefore, it is not clear how one can define measure entropy for G, but for any Borel probability measure μ there exists a local upper (resp., lower) measure entropy of G (in the sense of Brin and Katok). We obtain some upper and lower estimatimations of the topological entropy of G by local measure entropies ([3]). The theory of Carathéodory structures, introduced and studied by Pesin for a single map, allows us to express the topological entropy of G as an upper capacity of some Carathéodory structure of dynamical origin. Following Katok, for any Borel probability measure we define measure entropy of G. Applying the theory of Carathéodory structures, we obtain a partial variational principle, i.e. we prove that Katok's measure entropy of G is upper estimated by the topological entropy of G ([4]).

References

[1] A. Biś, M. Urbański, *Some remarks on topological entropy of a semigroup of continuous maps*, CUBO A Mathematical Journal 8 (2006), 63-71.

[2] A. Biś, *Entropies of a semigroup of maps*, Discrete and Cont. Dyn. Sys. 11 (2004), 639-648.

[3] A. Biś, *An analogue of the variational principle for group and pseudogroup actions*, Annales de l'Institut Fourier 63 (2013), 839-863.

[4] A. Biś, D. Dikranjan, A. Giordano Bruno and L. Stoyanov, *Topological entropy, upper capacity and fractal dimensions of finitely generated semigroup actins*, preprint.

The dimension spectrum of infinitely generated conformal dynamical systems
Vasileios Chousionis

The dimension spectrum of an iterated function system is the set of all possible values of the Hausdorff dimension of its subsystems. We perform a comprehensive study of the dimension spectrum of general conformal graph directed Markov systems modeled by countable state symbolic subshifts of finite type. As a corollary we show that the dimension spectrum of infinite conformal iterated function systems is compact and perfect. Our proofs depend on new topological pressure estimates for subsystems in the abstract setting of symbolic dynamics with countable alphabets.

We then focus our attention to conformal iterated function systems associated to real and complex continued fractions. According to the Texan conjecture, proven by Kesseböhmer and Zhu in 1996, the dimension spectrum of real continued fractions is full. We revisit the problem and we prove that if the alphabet of the continued fractions algorithm is any arithmetic progression, the set of primes, or the set of squares then the corresponding system has full dimension spectrum. On the way we obtain rigorous effective estimates for the Hausdorff dimension of continued fractions whose entries are restricted to infinite sets. Finally we show that the system resulting from the complex continued fractions algorithm has full dimension spectrum. Based on joint works with Dmitriy Leykekhman (UConn) and Mariusz Urbański (UNT).

Comments on conformal measures
Manfred Denker

I will review the notion of conformal measures from my perspectives: from its origin to recent results together with coauthors.

The space of ergodic measures for partially hyberbolic diffeomorphisms
Lorenzo Díaz

We study how nonhyperbolic ergodic measures appear in transitive nonhyperbolic settings. When the center direction is one-dimensional, the space of ergodic measures splits into three parts according to the central Lyapunov exponent: positive, zero, and negative. In the particular case when there is a central foliation by circles, we explain the structure of these three spaces and see how the measures with positive and negative Lyapunov exponents glue throughout the nonhypebolic ones.

Dynamics of fibered endomorphisms of $\mathbb{P}^2(\mathbb{C})$
Christophe Dupont

The talk concerns the endomorphisms of $\mathbb{P}^2(\mathbb{C})$ preserving a pencil of lines, those maps generalize the polynomial skew products of $\mathbb{C}^2$ studied by M. Jonsson. We show that the equilibrium measure decomposes (Fubini's formula relative to the invariant pencil) and we describe the Lyapunov exponents (formula using a relative Green function). In particular, we get endomorphisms of $\mathbb{P}^2(\mathbb{C})$ with a minimal exponent which are not suspensions of LattĐş maps on $\mathbb{P}^1(\mathbb{C})$. This is a joint work with J. Taflin.

Hausdorff, box-counting and intermediate dimensions
Kenneth Falconer

Firstly we will discuss an approach to box-counting dimensions based on capacities which leads easily to projection properties and other geometric results. Secondly we will discuss recent work with Fraser and Kempton on a continuum of dimensions which has Hausdorff dimension at one end of the range and box-counting dimension at the other.

Interval projections of self-similar sets
Abel Farkas

A 1-dimensional self-similar set on the plane may have infinitely many projections of positive Lebesgue measure. If the open set condition is satisfied then a projection has positive Lebesgue measure if and only if it contains an interval. In this talk we discuss the case when the projection is an interval. Under quite general conditions we show that only finitely many projection is an interval.

Exceptional sets in nonuniformly hyperbolic dynamics
Katrin Gelfert

We study exceptional sets, that is, the set of points whose orbits do not accumulate at a given "target set". We show that if the fractal dimension/entropy of the target is "sufficiently small" then the fractal dimension/entropy of the exceptional set is "full". Dynamical systems to which this type of results hold true are for example rational maps of the Riemann sphere and nonuniformly hyperbolic surface diffeomorphisms. Particular consequences occur when there is some a priori defined hyperbolic structure and, for example, if there exists an SRB measure. This is joint work with Sara Campos (UFJF).

TBA
Eugen Ghenciu

Critical points of the multiplier map
Igors Gorbovickis

The multiplier of a non-parabolic periodic orbit of a map $z^2 + c_0$ can be extended by means of analytic continuation to a multiple-valued algebraic function on the space of all quadratic polynomials $z^2 + c$. Information about the location of the critical points of this function might shed light on the question of possible shapes of hyperbolic components of the Mandelbrot set. We show that as the period of the periodic orbits increases to infinity, the critical points of the multiplier map equidistribute on the boundary of the Mandelbrot set. This is a joint work with Tanya Firsova.

TBA
Jacek Graczyk

On the measure-theoretic properties of coarse expanding maps
Peter Haïssinsky

The talk will be devoted to establish the basic metric properties of finite branched coverings which enjoy an expanding behavior.

Limiting Return Times Distribution for Invariant Measures
Nicolai Haydn

We consider a map acting on a space Ω that carries an invariant probability measure. For a positive measure set the return time is by Kac's theorem on average the reciprocal of the measure of the return set. We then take a nested sequence of positive measure sets which contract to a zero measure set Γ in the given space Ω, and show that if the return times distributions, when rescaled according to Kac's law, converge then the limit will be a compound Poisson distribution. The simplest case is when the limiting set Γ is a single point in which case on obtains a regular Poisson distribution if the limiting point is non-periodic and a Pólya-Aeppli distribution if the limiting point is periodic. We then apply this result to the synchronisation of coupled interval maps where the diagonal is an invariant limiting set Γ. We given an expression for the coefficients in the compound Poisson distribution which in general is not Pólya-Aeppli.

On the directional derivative of the Hausdorff dimension of quadratic polynomial Julia sets at 1/4

Ludwik Jaksztas

Let $d(\varepsilon)$ denotes the Hausdorff dimension of the Julia sets of the polynomials $f_\varepsilon(z) = z^2 + 1/4 + \varepsilon$.

We will study the directional derivative of the function $d(\varepsilon)$ along directions landing at the parameter 0, which corresponds to 1/4 in the case of family $z^2 + c$. We will consider all directions, except the one $\varepsilon \in \mathbb{R}^+$ which is outside the Mandelbrot set and is related to the parabolic implosion phenomenon.

We will see that for directions in the closed left half-plane the derivative of d is negative. Computer calculations show that it is negative except a cone (with opening angle approximately $150°$) around $\mathbb{R}^+$.

Self-conformal sets with positive Hausdorff measure

Antti Käenmäki

We show that any Hausdorff measurable subset of a self-conformal set has comparable Hausdorff measure and Hausdorff content. In particular, this proves that self-conformal sets with positive Hausdorff measure are Ahlfors regular, irrespective of separation conditions. When restricting to the real line and self-conformal sets with Hausdorff dimension strictly less than one, we additionally show that Ahlfors regularity is equivalent to the weak separation condition. In fact, we resolve a self-conformal extension of the dimension drop conjecture for self-conformal sets with positive Hausdorff measure by showing that its Hausdorff dimension falls below the expected value if and only if there are exact overlaps. The talk is based on joint work with Jasmina Angelevska and Sascha Troscheit.

Escaping points in the boundaries of Baker domains

Bogusława Karpińska

This talk concerns the dynamical behaviour of points in the boundaries of simply connected invariant Baker domains U of meromorphic maps with finite degree on U. We show that the set of boundary points that escape to infinity under iteration can have zero or full harmonic measure, depending on the type of Baker domain. Additionally we present some extensions to the infinite degree case. The talk is based on a joint work with Krzysztof Barański, Nuria Fagella and Xavier Jarque.

Multifractal Decompositions of Transient Dynamics

Marc Kesseböhmer

We develop a new thermodynamic formalism to investigate the transient behaviour of maps on the real line which are skew-periodic $\mathbb{Z}$-extensions of expanding interval maps. Our main focus lies in the dimensional analysis of the recurrent and transient sets as well as in determining the whole dimension spectrum with respect to the α-escaping sets. Our results provide a one-dimensional model for the phenomenon of dimension gaps which occur for limit sets of Kleinian groups. In particular, we are able to precisely quantify the height of the dimension drop in this setting. (Joint work with Maik GrÃűger and Johannes Jaerisch).

Characterization of finite PDEs, ODEs, and mixed integro-differential equations
Anton A. Kutsenko

We consider C^*-algebras $\mathscr{H}_{N,M}$ generated by N-dimensional differential operators with $M\times M$-matrix valued periodic coefficients. We show that the algebras $\mathscr{H}_{N,M}$ are all $*$-isomorphic to the universal uniformly hyperfinite algebra $\bigotimes_{n=1}^{\infty}\mathbb{C}^{n\times n}$ with the corresponding Glimm-Bratteli supernatural number $\mathfrak{n}(\mathscr{H}_{N,M})=\prod_{n=1}^{\infty}n$. The unitary transform between PDEs and ODEs has a fractal nature.

While the differential algebras $\mathscr{H}_{N,M}$ are insensitive to the dimensions $N,M\in\mathbb{N}$, the integro-differential algebras $\mathscr{F}_{N,M}$ are insensitive to the dimension M only. Their Glimm-Bratteli numbers are

$$\mathfrak{n}(\mathscr{F}_{N,M})=\prod_{n=1}^{\infty}\begin{pmatrix} n & 0\\ n-1 & 1\end{pmatrix}^{\otimes N}.$$

The effects of non-linear and stochastic terms will be also discussed. Some of the results are published in

[1] Anton A. Kutsenko (2019) Mixed multidimensional integral operators with piecewise constant kernels and their representations, *Linear and Multilinear Algebra*, **67**, 186-195

Measures of maximal entropy for suspension flows?
Tamara Kucherenko

We establish the existence of a suspension flow with continuous roof function which satisfies the following property. The set of measures of maximal entropy for the flow consists precisely of measures which maximize entropy on a prescribed invariant subset on the base. This result has a number of corollaries on how the set of measures of maximal entropy for the flow can be bad, even over a very nice space such as the full shift. (joint work with D. Thompson)

Transversality of hyperbolic and parabolic maps
Genadi Levin

We consider families of holomorphic maps defined on subsets of the complex plane, and show that the technique developed in [1] to treat unfolding of critical relations can also be used to deal with cases where the critical orbit converges to a hyperbolic attracting or a parabolic periodic orbit. As before this result applies to rather general families of maps, such as polynomial-like mappings, provided some lifting property holds. We prove that either the multiplier of a hyperbolic attracting periodic orbit depends univalently on the parameter and bifurcations at parabolic periodic points are generic, or one has persistency of periodic orbits with a fixed multiplier. As an application, we show that periodic points on the real line do not disappear after born for many families of real maps, from the real quadratic one to real sin-family and families with the flat critical point.

In the talk, I plan to outline the approach of [1] and its variation to the setting of hyperbolic and parabolic maps.

Joint work with Weixiao Shen and Sebastian van Strien.

References

[1] G. Levin, W. Shen and S. van Strien, *Monotonicity of entropy and positively oriented transversality for families of interval maps*, arXiv:1611.10056, 2016.
[2] G. Levin, W. Shen and S. van Strien, *Transversality in the setting of hyperbolic and parabolic maps*, arXiv:1901.09941, 2019.

Wandering domains for entire functions of finite order in the class $\mathcal{B}$
David Martí-Pete

Recently Bishop constructed the first example of a bounded-type transcendental entire function with a wandering domain using a new technique called quasiconfomal folding. It is easy to check that his method produces a function of infinite order. We construct the first examples of functions in the class $\mathcal{B}$ of finite order with wandering domains. In Bishop's example, as well as in our construction, the wandering domains are of oscillating type, that is, with an unbounded non-escaping orbit. To build such a function, we use quasiconformal interpolation instead of quasiconformal folding, which is much more straightforward. Our examples have order $p/2$ for any positive integer p and thus, since functions in the class $\mathcal{B}$ have order at least $1/2$, we can achieve the smallest possible order. This is a joint work with Mitsuhiro Shishikura.

Hausdorff dimension exceptional set estimates for projections, sections and intersections
Pertti Mattila

Let A and B be Borel sets in $\mathbb{R}^n$. If the Hausdorff dimension $\dim A \leq m$, then, by Marstrand's projection theorem, typical projections on m-planes preserve the dimension, and if $\dim A > m$, then typical projections of A have positive Lebesgue measure. Analogous basic results hold for radial projections, m-plane sections and intersections $A \cap (g(B) + z), g \in O(n), z \in \mathbb{R}^n$. In all cases there are estimates for the Hausdorff dimension of exceptional parameters. I review some old and recent such estimates.

Thermodynamic formalism and Ruelle's property for entire functions
Volker Mayer

We first present joint work with Mariusz Urbański in which we showed that, for a large class of entire functions, the full thermodynamic formalism holds provided the logarithmic tracts have some nice geometry. In order to do so, we introduce an integral means spectrum which takes care of the fractal behavior of the boundary of the tracts near infinity. It turns out that this spectrum behaves well as soon as the tracts have some sufficiently nice geometry which, for example, is the case for quasidisk or Hölder tracts.

One of its consequences is that we get analytic variation of the hyperbolic dimension for many families of entire functions. Contrary to that we observed recently with Anna Zdunik that this fails in general: there exists a holomorphic family of hyperbolic entire functions of finite order for which the variation of the hyperbolic dimension is not analytic.

Mixing and rates of mixing for finite and infinite measure flows
Ian Melbourne

We will describe various results on mixing for flows that can be viewed as a suspension over a Gibbs-Markov map (modulo contracting directions). The roof function may be integrable or nonintegrable.

An example is the infinite horizon planar periodic Lorentz gas where the mixing rate is 1/t and we obtain sharp upper (joint with Bálint & Butterley) and lower (joint with Bruin & Terhesiu) bounds for this rate.

Dynamical lifts over iterated systems with overlaps
Eugen Mihailescu

In this talk we present certain dynamical lifts over conformal iterated function systems $\mathcal{S}$ which do not satisfy Open Set Condition. We introduce notions of overlap functions and overlap numbers for equilibrium measures over the limit set Λ. Overlap numbers are proved to be connected to the folding entropies of measures for endomorphisms. Moreover they provide thermodynamical estimates for the dimensions of projection measures on Λ. In particular topological overlap numbers are studied in some cases. Based on joint work with Mariusz Urbański.

Isometry of trees for polynomials
Hongming Nie

Let $\{P_t\}$ be a degenerate meromorphic family of degree $d \geq 2$ polynomials. DeMarco and McMullen constructed a limiting tree for $\{P_t\}$ by considering the level sets of escaping rate. Also, following Kiwi and Trucco, there is an induced map acting on a suitable Berkovich space and a natural Berkovich tree associated to $\{P_t\}$. In this talk, I will show that associated with suitable metrics, these two trees for $\{P_t\}$ are isometric.

Expected Hausdorff dimension of symmetric Cantor sets
Piotr Nowakowski

We consider the family $\mathcal{CS}$ of symmetric Cantor subsets of $[0,1]$. Each set in $\mathcal{CS}$ is uniquely determined by a sequence $a = (a_n)$ belonging to the Polish space $X := (0,1)^{\mathbb{N}}$ that is equipped with the probabilility product measure μ generated by Lebesgue measure on $(0,1)$. This yields a one-to-one correspondence between sets in $\mathcal{CS}$ and sequences in X. If $\mathcal{A} \subset \mathcal{CS}$, the corresponding subset of X is denoted by $\mathcal{A}^*$. We study the subfamilies $\mathcal{H}_s$ of $\mathcal{CS}$, consisting of sets with Haudsdorff dimension s for $s \in (0,1)$. We prove that the set $\mathcal{H}_0^*$ is residual in X, and

$$\mu(\mathcal{H}_s^*) = \begin{cases} 1, \ s = \frac{\ln 2}{\ln 2+1} \\ 0, \ s \neq \frac{\ln 2}{\ln 2+1} \end{cases} .$$

The results were obtained together with Marek Balcerzak and Tomasz Filipczak.

Commuting rational functions revisited
Fedor Pakovich

Let A and B be rational functions on the Riemann sphere. The classical Ritt theorem states that if A and B commute and do not have an iterate in common, then up to a conjugacy they are either powers, or Chebyshev polynomials, or Lattès maps. This result however provides no information about commuting rational functions which *do* have a common iterate. On the other hand, non-trivial examples of such functions exist and were constructed already by Ritt. In the talk we present new results concerning this class of commuting rational functions. In particular, we describe a method which permits to describe all rational functions commuting with a given rational function.

Long hitting times for expanding systems
Łukasz Pawelec

We will take a brief look into quantitative recurrence, i.e. the behaviour of the return time map $\tau_U(x)$. Currently, there is a lot of papers showing that the recurrence in dynamical systems is often faster than the Kac's Lemma would suggest. We will provide an opposite counterpart to these results by proving that for many common systems the hitting times into shrinking balls are also often much larger then expected, by showing that for many dynamical systems

$$\limsup_{r\to 0} \tau_{B(y,r)}(x)\mu(B(y,r)) = +\infty,$$

for all y and almost all x.

This is joint work with Mariusz Urbański.

Graph energies and Ahlfors-regular conformal dimension
Kevin M. Pilgrim

Let f be a hyperbolic rational function with connected Julia set J. The dynamics of the restriction $f : J \to J$ may be faithfully encoded by a pair of maps $\pi, \phi : \Gamma_1 \to \Gamma_0$ between finite planar graphs, well-defined up to homotopy. We relate the Ahlfors-regular conformal dimension $\mathrm{ARconfdim}(J)$ of J to a critical exponent $1 \leq p_* < 2$ for an asymptotic dynamical energy $\overline{E}^p[\pi, \phi]$ associated to $\pi, \phi : \Gamma_1 \to \Gamma_0$. Applied to concrete examples, our methods and hand calculations yield nontrivial lower and upper bounds for $\mathrm{ARconfdim}(J)$. Computer implementations give finer empirical estimates. This is joint ongoing work with D. Thurston.

Countable α-Kakutani sequences
Mark Pollicott

Let $0 < \alpha < 1$ and consider the division of the unit interval $[0, 1]$ into the subintervals $[0, \alpha]$ and $[\alpha, 1]$. Choose the largest of these two intervals and further subdivide it into two smaller subintervals whose lengths are in the same ratio (i.e., $\alpha : 1 - \alpha$). Next choose the largest of these three intervals and subdivide it into two smaller subintervals whose lengths are again in the same ratio. Iterating this construction gives sequences of consecutively largest remaining intervals at each step. Kakutani showed that the sequence of their left end points (say) is uniformly distributed. We consider a more general problem where we begin with a partition of $[0, 1]$ into a family of countably many intervals (of lengths $0 < \alpha_i < 1$ with $\sum_{i=1}^{\infty} \alpha_i = 1$). We give conditions for the natural analogue of the Kakutani result on uniform distribution. This is joint work with Ben Sewell.

Multifractal Hausdorff dimensions for invariant subsets of a piecewise monotonic map on the interval
Peter Raith

Suppose that $T : [0, 1] \to [0, 1]$ is a piecewise monotonic map, this means there exists a finite partition $\mathcal{Z}$ into pairwise disjoint open intervals with $\bigcup_{Z \in \mathcal{Z}} \overline{Z} = [0, 1]$ such that $T\big|_Z$ is continuous and strictly monotonic for all $Z \in \mathcal{Z}$. Moreover it is assumed that for all $Z \in \mathbb{Z}$ the map $T\big|_Z$ is differentiable and its derivative can be extended to a continuous function on the closure of Z. For a finite union U of open intervals set $A := [0, 1] \setminus \bigcup_{j=0}^{\infty} T^{-j} U$, which is the set of all points whose orbits omit U. It is assumed that the restriction of T to A has positive topological entropy.

Results on multifractal Hausdorff dimensions of A are presented. At first a formula on the multifractal Hausdorff dimension and multifractal packing dimension of an ergodic invariant measure with positive entropy is stated. Then one obtains a formula for the essential multifractal Hausdorff dimension and the essential multifractal packing dimension. Finally, in the case of a certain expansiveness, one gets also a result on the multifractal Hausdorff dimension.

Mass Transference Principle for sets of arbitrary shapes
Michał Rams

The Beresnevich and Velani's Mass Transference Principle is a classical result in the geometric measure theory. It says that if we have a collection of geometric balls $\{B_i\}$ in the d-dimensional Euclidean space and the limsup set of those balls has full Lebesgue measure then we can get a nontrivial lower bound on the Hausdorff dimension of the limsup set of the family of balls $\{E_i\}$ with the same centers but with smaller radii $|E_i| = |B_i|^a$.

This very useful result turns out to be surprisingly difficult to generalize. Even a most delicate modification – allowing for the limsup set to be not of full Lebesgue measure but of full Hausdorff dimension – can lead to a catastrophe. There exists a theorem by Allen and Baker generalizing the Mass Transference Principle to a more general setting, but it involves very strong additional assumptions on the geometry of the picture.

However, there exists one direction in which the Mass Transference Principle can be generalized without great effort. Wang, Wu and Xu proved a version of Mass Transference Principle in which the sets E_i are not balls but ellipsoids, with semiaxes of E_i taking values $|B_i|^{a_1}, \ldots, |B_i|^{a_d}$. In this talk I will present a further generalization, in which E_i can be any open sets, with each E_i contained in the corresponding B_i. This is a joint work with Henna Koivusalo from Universitat Wien.

Classifying simply connected wandering domains
Philip Rippon

For rational functions, a classification of periodic Fatou components, with a detailed description of the dynamical behaviour inside each of the four possible types, was given around 100 years ago. For transcendental entire functions, there is an additional type known as Baker domains, which are now well understood, and many examples of wandering domains, which cannot occur for rational functions. Recently a detailed description of the dynamical behaviour inside multiply connected wandering domains has been given, but there has been no comparable study of simply connected wandering domains. We give a classification into different types in terms of the hyperbolic distance between iterates and whether orbits approach the boundaries of the wandering domains, and show that all such types are realisable. This is joint work with A.M. Benini, V. Evdoridou, N. Fagella and G. Stallard.

Dimension estimates for the attractors of some non-conformal C^1 Iterated Function Systems
Károly Simon

(Joint work with De-Jun Feng)

Given a finite list $\mathcal{F} = \{f_1, \ldots, f_m\}$ of contracting, non-conformal $\mathcal{C}^1$ diffeomorphisms $f_i : \mathbb{R}^d \to \mathbb{R}^d$, for a $d \geq 2$. We give an upper bound on the upper box dimension of the attractor Λ. On $\mathbb{R}^2$, we prove an almost all-type result which says that the previously mentioned upper bound is actually the Hausdorff dimension of Λ under some conditions.

Dimensions of Julia and escaping sets of transcendental entire functions
Gwyneth Stallard

Many results have been proved on the possible values of the dimensions and areas of the Julia and escaping sets of transcendental entire functions. In this talk we consider the different values that can occur depending on whether or not a function belongs to the much studied Eremenko-Lyubich class and discuss interesting open questions.

Zero-dimensional compact metrizable spaces as attractors of (generalized) iterated function systems
Filip Strobin

In the last years, the problem of considering zero-dimensional compact metrizable spaces as attractors of iterated function systems has been undertaken by several authors.

I will show that each such a space X which is uncountable or is countable but has successor scattered height, can be embedded into the real line $\mathbb{R}$ as the attractor of a strictly contracting IFS (this result was obtained jointly with T. Banakh and M. Nowak, and, independently, by E. D'Aniello and T.H. Steele), whereas if X is countable with limit scattered height, then it is not homeomorphic to the attractor of any weakly contracting IFS (this fact was proved by

M. Nowak). Finally, I will present a result obtained jointly with Ł. Maślanka saying that in all cases, X can be embedded into the real line as the attractor of a *generalized* IFS in the sense of A. Mihail and R. Miculescu. In particular, this shows that the class of attractors of generalized IFSs is essentially wider than the class of classical IFSs' attractors. I will also recall some other results in this setting.

Similarity Map for Harmonically Generic Parameters
Grzegorz Świątek
(Joint work with Jacek Graczyk)

We study conformal quantities at generic parameters with respect to the harmonic measure on the boundary of the connectedness loci $\mathcal{M}_d$ for unicritical polynomials $f_c(z) = z^d + c$. It is known that these parameters are structurally unstable and have stochastic dynamics. We prove $C^{1+\frac{\alpha}{d}-\epsilon}$-conformality, $\alpha = \mathrm{HD}\,(\mathcal{J}_{c_0})$, of the parameter-phase space similarity maps $\Upsilon_{c_0}(z) : \mathbb{C} \mapsto \mathbb{C}$ at typical $c_0 \in \partial\mathcal{M}_d$ and establish that globally quasiconformal similarity maps $\Upsilon_{c_0}(z)$, $c_0 \in \partial\mathcal{M}_d$, are C^1-conformal along external rays landing at c_0 in $\mathbb{C}\setminus\mathcal{J}_{c_0}$ mapping onto the corresponding rays of $\mathcal{M}_d$. This conformal equivalence leads to the proof that the z-derivative of the similarity map $\Upsilon_{c_0}(z)$ at typical $c_0 \in \partial\mathcal{M}_d$ is equal to $1/\mathcal{T}'(c_0)$, where $\mathcal{T}(c_0) = \sum_{n=0}^{\infty}(D(f_{c_0}^n)(c_0))^{-1}$ is the transversality function.

Entropy and drift for Gibbs measures on geometrically finite manifolds
Giulio Tiozzo
(Joint work with I. Gekhtman)

The boundary of a simply connected, negatively curved manifold carries two natural types of measures: on one hand, conditionals of Gibbs measures such as the Patterson-Sullivan measure and the SRB measure. On the other hand, harmonic measures arising from random walks.

We prove that the absolute continuity between a harmonic measure and a Gibbs measure is equivalent to a relation between entropy, drift and critical exponent, extending the previous formulas of Guivarc'h, Ledrappier, and Blachère-Haïssinsky-Mathieu. This shows that if the manifold (or more generally, a CAT(-1) space) is geometrically finite but not convex cocompact, harmonic measures are singular with respect to Gibbs measures.

Escape rates for multimodal maps with holes
Mike Todd

I'll present recent work with Mark Demers on multimodal interval maps with holes. We make various generalisations of the current results on this topic, allowing the hole to be placed anywhere in the interval and getting escape rates, conditionally invariant measures and a variational principle, even when critical points have dense orbits, and with no slow approach conditions required. The measures we deal with are equilibrium states, either for Hölder potentials or for 'geometric potentials', so, for example, we include acips here in the Collet-Eckmann case. Under an extra (though much weaker than previously) assumption, we also prove 'zero-hole limits'.

The Ruelle property: Old and new.
Michel Zinsmeister & Huo Shengjin

By Bers famous theorem, two quasiconformally equivalent Riemann surfaces can be simultaneously uniformized by a quasifuchsian group: if S is a Riemann surface we can thus define an application from $\mathcal{T}(S) \times \mathcal{T}(S)$ to R_+ which associates to each pair of elements of the Teichmuller space $\mathcal{T}(S)$ the dimension of the limit set of the associated quasifuchsian group. A famous result by Ruelle asserts that this map is real-analytic if S is compact: we say that compact surfaces have Ruelle property. What are the Riemann surfaces that share this Ruelle's property? The aim of this talk is to survey about what is known and not known about this problem: we will mainly discuss old results and a few new ones.

Hitting times and positions in rare events
Roland Zweimüller

I will present abstract limit theorems which provide sufficient conditions for a sequence (A_l) of rare events (sets with $\mu(A_l) \to 0$) in an ergodic probability preserving dynamical system to exhibit Poisson asymptotics, and for the consecutive positions inside the A_l to be asymptotically iid (spatiotemporal Poisson limits). The limit theorems only use information on what happens to A_l before some time τ_l which is of order $o(1/\mu(A_l))$. In particular, no assumptions on the asymptotic behavior of the system akin to classical mixing conditions are used.