DYNAMICAL PROPERTIES RELATED TO THE BESICOVITCH PSEUDO-METRIC

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Abstract

A topological dynamical system is a pair (X, T) consisting of a compact metric space X and a continuous self-map T of X. The Besicovitch pseudometric provides a different way of measuring the "closeness" of two orbits of a topological dynamical system that focuses on long-term behavior. This pseudometric is particularly useful for studying the statistical properties of orbits, especially invariant measures generated by these orbits. For symbolic systems (subshifts) the Besicovitch pseudometric is uniformly equivalent to the pseudometric $\bar{d}$ that measures the asymptotic density of differences between two sequences of symbols. During my talk, I will present the results on the Besicovitch and $\bar{d}$ pseudometrics obtained in the course of my PhD studies and contained in the article [2], and in preprints [1, 3]. We study limits of sequences of generic points with respect to the Besicovitch pseudometric D_B. Using the characterization of the spectrum of an ergodic measure via its generic points in [1], we analyze the properties of the (limit) measure by passing to the limit in D_B. Specifically, we show that the set of generic points for discrete spectrum, totally ergodic, (weakly) mixing, and zero entropy measures forms a closed set with respect to the Besicovitch pseudo-metric. We also discuss joint results on the pseudo-metric $\bar{d}$ on shift spaces that are presented in [2]. We also study connections between the asymptotic average shadowing property (a variant of the classical shadowing property) and the vague specification property (a variant of the classical specification property) for general topological dynamical systems. Using the Besicovitch pseudo-metric and its relatives we show that the asymptotic average shadowing property and the vague specification property are equivalent.

References

[1] S. Babel, M. Can, D. Kwietniak, P. Oprocha, *Spectrum of invariant measures via generic points*, preprint.

[2] M Can, J. Konieczny, M. Kupsa, and D. Kwietniak, *Minimal and proximal examples of $\bar{d}$-stable and $\bar{d}$-approachable shift spaces*, Ergodic Theory and Dynamical Systems, 2025;45(2):396-426. doi:10.1017/etds.2024.43.

[3] M. Can, A. Trilles, *The equivalence of asymptotic average shadowing and vague specification properties and its consequences*, [arXiv:2411.01556], preprint.