The Nambu-determinant Poisson brackets on $\mathbb{R}^d$ are expressed by the formula

$$\{f,g\}_d(\boldsymbol{x}) = \varrho(\boldsymbol{x}) \cdot \det\bigl(\partial(f,g,a_1,...a_{d-2})/\partial(x^1,...,x^d)\bigr),$$

where $a_1, \ldots, a_{d-2}$ are smooth functions and $x^1, \ldots, x^d$ are global coordinates (e.g., Cartesian), so that $\varrho(\boldsymbol{x}) \cdot \partial_{\boldsymbol{x}}$ is the top-degree multivector.

For an example of Nambu–Poisson bracket in classical mechanics, consider the Euler top with $\{x,y\}_3 = z$ and so on cyclically on $\mathbb{R}^3$.

Independently, Nambu's binary bracket $\{-,-\}_d$ with Jacobian determinant and $d-2$ Casimirs $a_1, \ldots, a_{d-2}$ belong to the Nambu (1973) class of N-ary multi-linear antisymmetric polyderivational brackets $\{-,\ldots,-\}_d$ which satisfy natural N-ary generalizations of the Jacobi identity for Lie algebras.

In the study of Kontsevich's infinitsimal deformations of Poisson brackets by using 'good' cocycles from the graph complex, we detect case-by-case that these deformations preserve the Nambu class, and we observe new, highly nonlinear differential-polynomial identities for Jacobian determinants over affine manifolds. In this talk, several types of such identities will be presented.

(Work in progress, joint with M. Jagoe Brown, F. Schipper, and R. Buring: see [arXiv:2112.03897] and [arXiv:2409.18875, 2409.12555, 2409.15932, 2503.10916, 2503.10926]; special thanks to the Habrok high-performance computing cluster.)