Foundations of Mathematics and Set Theory

(Department of Foundations of Mathematics, Department of Algebra)

Results:

1. Jerzy Łoś in the paper "Quelques remarques, théorèmes et problèmes sur les classes définissables d'algèbres" in: Mathematical Interpretations of formal systems, Amsterdam 1955, pp. 98–113, introduced ultraproducts of relational structures and proved the fundamental theorem:

   $$\prod_{i\in I} \mathfrak{A}_i \bmod F \models \varphi \;\equiv\; \{i \in I :\; \mathfrak{A}_i \models \varphi\} \in F$$

   (the formula φ is true in the ultraproduct F, if and only if it is true on F-almost every axis).
   This theorem, called also the Łoś lemma, is present in almost every handbook of mathematical logic or model theory and has numerous applications in model theory and algebra.
2. Andrzej Mostowski in the paper "Models of axiomatic theories admitting automorphisms", Fund. Math. 43 (1957), pp. 50–68, with Andrzej Ehrenfeucht proved the theorem:
   A theory having an infinite model has a model $\mathfrak{A}$ which contains a given infinite linear ordering $(\mathfrak{A}, \leq)$ in such a way that increasing sequences $\langle a_1, \ldots, a_n\rangle$ (for each $n \in \mathbb{N}$) satisfy in $\mathfrak{A}$ the same formulas.
   This theorem made a spectacular career. It started the investigation of order indiscernible elements (indiscernibles), had various modifications, generalizations and applications in set theory, model theory and logic, and also in algebra (indiscernibles generalize free generators).

3. Jan Mycielski in the paper ”A mathematical axiom contradicting the axiom of choice”, Bull. Polon. Acad. Sci. 10 (1962), pp. 1–3, with Hugo Steinhaus introduced the so called axiom of determinacy. This axiom states the existence of a winning strategy in some simple games determined by sets of reals. The full axiom of determinacy (every game is determined) is inconsistent (proved already by the authors) with the axiom of choice, however the determinacy property itself when restricted to some families of sets (e.g. to the family of open sets, Borel sets etc.) is an interesting property of the sets of the family (implies for instance measurability and the Baire property, which was proved later by Mycielski and Świerczkowski), which was investigated in the next years by numerous top logicians (Woodin, Martin, Solovay).

Monographs:

1. A. Mostowski, *Sentences undecidable in formalized arithmetic*, North-Holland 1952.
2. W. Sierpiński, *Cardinal and ordinal numbers*, PWN 1958, 2nd ed. 1965.
3. R. Sikorski, *Boolean Algebras*, Springer 1960, 2nd ed. 1964.
4. H. Rasiowa & R. Sikorski, *Mathematics of metamathematics*, PWN 1963, 2nd ed. 1968.
5. A. Mostowski, *Thirty years of foundational studies*, Helsinki 1965.
6. K. Kuratowski & A. Mostowski, *Set theory*, North-Holland 1967, 2nd ed. 1976.
7. A. Mostowski, *Constructible sets with applications*, North-Holland 1969.

Number Theory (Department of Number Theory)

Results:

1. (Henryk Iwaniec, 1970–1980) Rediscovery and justification of the variant of the sieve method proposed earlier by Rosser but never published by him, known now as the Rosser-Iwaniec sieve (*Rosser's sieve*, Acta Arith. 36, 171–202).
2. (Henryk Iwaniec, 1974) Theorem:

   Every quadratic polynomial with integer coefficients depending essentially on two variables, irreducible, primitive and representing arbitrarily large odd numbers represents infinitely many primes. (*Primes represented by quadratic polynomials in two variables*, Acta Arith. 24, 435–459)
3. (Henryk Iwaniec, 1978) Theorem:

   Every quadratic univariate polynomial irreducible and without any fixed divisor > 1 *represents infinitely many numbers with at most two prime factors counting multiplicities.* (*Almost primes represented by quadratic polynomials*, Invent. Math. 47, 171–188)

Monographs:

1. W. Sierpiński, *Elementary Theory of Numbers*, PWN 1964, 2nd ed. 1987.
2. A. Rotkiewicz, *Pseudoprime numbers and their generalizations*, Novi Sad 1972.
3. A. Schinzel, *Polynomials with special regard to reducibility*, Encyclopedia of Mathematics and its Applications 77, Cambridge University Press 2000.

4. J. Urbanowicz & K. S. Williams, *Congruences for L-functions*, Kluwer Academic Publishers, Dordrecht/Boston/London, 2000.

Algebra and Algebraic Geometry (Department of Algebra)

Results:

1. (Edward Sąsiada, 1967) Construction of the first example of a simple ring with nonzero multiplication and radical in the sense of Jacobson. (E. Sąsiada & P. M. Cohn, *An example of a simple radical ring*, J. Algebra 5, 373–377)
2. (Andrzej Białynicki-Birula, 1973) Theorem about decomposition:
   Suppose that a one dimensional algebraic torus acts on a smooth complete algebraic variety. Then there exists a unique equivariant decomposition of the variety into a union of equivariant fibre bundles over the components of the set of the fixed points with fibers isomorphic to vector spaces equipped with linear actions of the torus. (*Some theorems on actions of algebraic groups*, Annals of Math. 98, 480–497)
3. (Piotr Pragacz, 1988) Description of the ideal of polynomials supported on a degeneracy locus of a universal bundle map in terms of an explicit set of generators and $\mathbb{Z}$-basis, with applications to the Chern numbers and Chow groups of determinantal varieties. (*Enumerative geometry of degeneracy loci*, Ann. scient. Éc. Norm. Sup. 21, 413-454)

Monographs:

1. S. Balcerzyk, *Introduction to homological algebra*, PWN 1972 (in Polish).
2. S. Balcerzyk & T. Józefiak, *Commutative Noetherian and Krull rings*, PWN 1989.

3. S. Balcerzyk & T. Józefiak, *Commutative rings. Dimension, multiplicity and homological methods*, PWN 1989.

Measure Theory

(former Department of of Mathematical Analysis)

Monograph:

J. Czyż, *Paradoxes of measures and dimensions oryginating in Felix Hausdorff's ideas*, World Scientific, Singapore, 1994.

Analytic functions

(former Department of Complex Analysis and Differential Geometry)

Results:

1. (Franciszek Leja, Zygmunt Charzyński, Mieczysław Biernacki, 1954–56) Founding of the contemporary theory of extremal polynomials for analytic functions of one variable. (F.L., *Polynômes extrémaux et la répresentation conforme des domaines doublement connexes*, Ann. Polon. Math. 1, 13–28, Z.C., *Sur les fonctions univalentes algébriques bornées*, Dissert. Math. 10, 39 pp., M.B., *Sur les polynômes dont tous les zéros sont réels*, Ann. Univ. Mariae Curie-Skłodowska Sect. A 10, 61–75)
2. (Józef Siciak, 1964) A generalization of Schwarz's lemma and of Hadamard's three circles theorem to analytic functions of several complex variables. (*A generalization of Schwarz's lemma and of Hadamard's three circles theorem*, Colloq. Math. 11, 203–207)

Differential Equations (Department of Differential Equations)

Results:

1. Tadeusz Ważewski presented his *topological principle* concerning asymptotic behaviour of trajectories of ordinary differential equations. The topological principle provides simple conditions which guarantee existence of trajectories which stay in a given open set T for all positive time interval (possibly infinite) in which they are defined. To be more precise, consider a system Σ of time dependent ordinary differential equations $\dot{x} = f(t, x)$ in an open domain $D \subset \mathbb{R} \times \mathbb{R}^n$. Assume that f is continuous and the Cauchy problem has unique solutions. The idea of the topological principle of Ważewski can be explained with the following special case.

   Given an open subset $T \subset D$, if the set S of exit points (from T to D) of trajectories of Σ coincides with the set of strict exit points and S is not a retract of T then there exists a trajectory in T which never leaves T to D.

   A general version of the topological principle was published in 1947 and its applications were presented in following papers and a lecture at the Amsterdam Congress of Mathematicians in 1954. It was one of the first major achievements (followed by ideas of C. Conley) in the applications of topology to the theory of ordinary differential equations, after the Second World War. ([1] T. Ważewski., *Sur un principle topologique de l'examen de l'allure asymptotique des intégrales des équations différentielles ordinaires*, Ann. Soc. Polon. de Math., T. XX (1947), [2] T. Ważewski, *Sur les intégrales d'un système des équations différentielles tangentes aux hyperplans caractéristiques issues du point singulier*, ibidem, T. XXI (1948), [3] T. Ważewski, *Sur l'évaluation du nombre des paramètres essentiels dont dépend la famille des intégrales d'un système des équations différentielles tangentes aux hyperplans caractéristiques issues du point singulier*, Bull. Acad. Polon. des Sciences, T. I, No. 1–2 (1953), [4] T. Ważewski, *Sur une méthode topologique de l'examen de l'allure asymptotique des intégrales des équations différentielles*, Proc. Int. Congress of Math., Amsterdam 1954, Vol. 3, pp. 132–139 (1956))

2. Bogdan Bojarski has been investigating the theory of Sobolev spaces and their applications to nonlinear PDE's in numerous profound papers written over many years. Together with Tadeusz Iwaniec, he laid out [1] a systematic analytic theory of quasiconformal mappings in $\mathbb{R}^n$. Later on, this work has become one of the fundamental references in quasiconformal analysis. His paper with Hajłasz [2], containing an important idea of pointwise approximation and giving various characterizations of Sobolev spaces in terms of maximal operators, paved the way to a very active research programme in analysis on metric spaces (J. Cheeger, J. Henonen, L. Capogna, J. Tyson, N. Shanmugalingam). Finally, in his recent works, see e.g. [3], he gives a description of functions in higher order Sobolev spaces in terms of their formal Taylor remainders $R^{m-1}f(x,y)$ which have to vanish like $|x-y|^m(a(x)+a(y))$, with the coefficients a in L^q. He applies these ideas to obtain a general Luzin-like theorem, characterizing those measurable functions that can be changed on a set of an arbitrarily small measure to obtain a function of class C^k. ([1] B. Bojarski and T. Iwaniec, *Analytical foundations of the theory of quasiconformal mappings in R^n*, Ann. Acad. Sci. Fenn. Ser. A I Math. 8 (1983), no. 2, 257–324, [2] B. Bojarski and P. Hajłasz, *Pointwise inequalities for Sobolev functions and some applications*,

Studia Math. 106 (1993), no. 1, 77–92, [3] B. Bojarski, *Pointwise characterization of Sobolev classes*, Proc. Steklov Inst. Math. no. 4 (255), 2006, 65–81)

3. (Andrzej Pliś, 1954, 1961) In the middle of 50-ties (in XX century) Andrzej Pliś created a technique which enabled him to construct examples of linear partial differential equations with smooth (C^∞) coefficients, for which the Cauchy problem has non-unique solutions [1]. As stated by Hörmander in his 4 volume monograph, this result was a great surprise to specialists. It was extended later by P. Cohen and by Hörmander himself. In a later work [2] Pliś constructed a linear elliptic differential operator P of order 4 on $\mathbb{R}^3$, with smooth coefficients, such that the equation $Pu = 0$ has a non-trivial solution on $\mathbb{R}^3$ with compact support. These results were presented at the International Congress of Mathematicians in Stockholm. ([1] A. Pliś, *The problem of uniqueness for the solution of a system of partial differential equations*, Bull. Acad. Pol. Sci. 2, 55–57, [2] A. Pliś, *A smooth linear elliptic differential equation without any solution in a sphere*, Comm. Pure Appl. Math. 14, 599–617)

4. (Bogdan Ziemian, 1988, 1994) Using the theory of complex integral transfoms and the theories of distributions and hyperfunctions, introduced a new class of generalized functions [1]. In his theory the Taylor expansion of an analytic function, in terms of functions x^i, $i \in \mathbb{Z}_+$, is replaced with an expansion of a generalized function as an integral of the (weighted) family of functions x^α, $\alpha \in \mathbb{R}_+$. He applied such functions for studying differential equations with singularities. For a broad class of partial differential operators he found new integral formulas for the fundamental solutions [2], where the integration takes place over subsets contained in the complex characteristic set of the

operator. These integral representations generalize formulas obtained earlier by Ehrenpreis, as well as the Leray residuum formula, in the case where the cycle intersects the singularity of the integrated function. They make possible investigation of solutions of differential equations at infinity. ([1] B. Ziemian, *Taylor formula for distributions*, Dissertationes Math. 264, 1–59, [2] B. Ziemian, *Leray residue formula and asymptotics of solutions to constant coefficients PDE's*, Topological Methods in Nonlinear Analysis 3, 257–293)

Dynamical Systems (Department of Dynamical Systems)
(Department of Functional Analysis) (former Department of Generalized Functions)

Results:

1. (Feliks Przytycki with Mariusz Urbański and Anna Zdunik, 1989–1991, 2006) Theorem:
Let U be a connected, simply connected open domain in the Riemann sphere $\bar{C}$, being the immediate basin of attraction to a fixed point of a holomorphic function defined on a neighbourhood of the closure of U. Then the following dichotomy takes place: Either ∂U is an analytic Jordan curve or an analytic arc, or its (hyperbolic) Hausdorff dimension is bigger than 1 (i.e. ∂U is fractal), harmonic measure ω on ∂U, viewed from U, is singular with respect to the first Hausdorff measure and moreover ω is "comparable" with the Hausdorff measure with the gauge function $t \mapsto t \exp\sqrt{c \log 1/t \log\log\log 1/t}$ for a non-zero constant c. If $\phi : D \to U$ is a univalent function from the unit disc D onto U, then for Lebesgue a.e. $\zeta \in \partial D$
$$\limsup_{t \nearrow 1} \frac{|\phi'(t\zeta)|}{\sqrt{\log 1/(1-t) \log\log\log 1/(1-t)}} = c \neq 0.$$
This c is related with σ^2, the asymptotic variance of a stationary stochastic process related to ϕ. (The references include *Harmonic, Gibbs and Hausdorff measures on repellers for holomorphic maps, I*, Ann. of Math. 130 (1989), 1–40; II, Studia Math. 97 (1991), 189–225)

2. (Feliks Przytycki (coauthors chronologically: S. Rohde, Tomasz Nowicki, S. Smirnov, J. Rivera-Letelier, Michał Rams, K. Gelfert), 1996–2009) Theorem:

Let $f : \bar{C} \to \bar{C}$ be a rational function. Then the property of being nonuniformly hyperbolic, more precisely Collet-Eckmann property (CE): $|(f^n)'(f(c))| \to \infty$ exponentially fast as $n \to \infty$, for every critical point c in Julia set $J(f)$, in presence of only one critical point in $J(f)$, is equivalent to so-called Topological Collet-Eckmann property (TCE), which is a topological conjugacy invariant.

Also for unimodal maps of interval with negative Schwarzian derivative, Collet-Eckmann property was proved to be a topological invariant. In presence of more critical points in $J(f)$ the property TCE was proved to be substantially weaker than CE. Several properties equivalent to TCE were introduced, in the language of Gibbs states, Lyapunov exponents, periodic orbits, strict decreasing of the pressure as the function of "temperature" $1/t \mapsto P(t) = P(f, -t\log|f'|)$. Analyticity of $P(t)$ was proved, on the maximal open interval where it is not affine, in a new class of functions f having "nice inducing", larger than TCE. By Legendre transform this yields the analyticity of the dimension spectrum of Lyapunov exponents

$$t \mapsto HD(\{z \in J(f) : \lim_{n\to\infty} \frac{1}{n} \log|(f^n)'(z)| = t\})$$

and the analyticity of the dimension spectrum of the local dimensions of maximal measure m

$$t \mapsto HD(\{z \in J(f) : \lim_{r \searrow 0} \frac{\log m(B(z,r))}{\log r} = t\}).$$

(The references include: Fund. Math. 155 (1998); Inventiones Mathematicae 151 (2003); Annales Scientifiques de l'École Normale Superieure 40 (2007).)

3. Andrzej Lasota in cooperation with J. A. Yorke, M. C. Mackey and his students among others Józef Myjak, Ryszard Rudnicki and Tomasz Szarek worked out methods of investigation of Markov operators and presented their applications in dynamical systems, theory of fractals and in biological and physical models (these investigations were initiated by Prof. Lasota in 1973 and they are continued till now).

Theory of Approximations

(Department of Probability Theory, Department of Functional Analysis)

Definition. The Franklin orthonormal system $(f_0, f_1, \ldots)$ on the interval $I = [0, 1]$ is defined as follows (Ph. Franklin, 1928):

$$f_0 = 1, \quad f_1(t) = 2\sqrt{3}(t - 1/2),$$

and to every $n \geq 2$, $n = 2^j + k$, $1 \leq k \leq 2^j$ we order the system of $n + 1$ nodes

$$t_{i,n} = \begin{cases} \dfrac{i}{2^{j+1}} & \text{for } i = 0, \ldots, 2k \\ \dfrac{i-k}{2^j} & \text{for } i = 2k+1, \ldots, n. \end{cases}$$

Now the function f_n is defined as a spline with the nodes $(t_{i,n}$, $i = 0, \ldots, n)$ normalized in $L^2(I)$, orthogonal to $f_0, f_1, \ldots, f_{n-1}$ such that $f_n(t_n) > 0$, where $t_n = t_{2k-1,n}$. We put

$$K_n(s, t) = \sum_{i=0}^{n} f_i(s) f_i(t).$$

Results:

1. (Zbigniew Ciesielski, 1966) Theorem:
   There exists an absolute constant $C > 0$ *such that for* $q = 2 - \sqrt{3}$ *and* $1 \le p \le \infty$

$$|K_n(t,s)| \le Cnq^{\frac{n}{2}|t-s|} \text{ for } s,t \in I;\; n = 1,2,\dots,$$

$$|f_n(t)| \le C\sqrt{n}q^{\frac{n}{2}|t-t_n|} \text{ for } t \in I;\; n = 1,2,\dots,$$

$$\left\| \sum_{n=2^j+1}^{2^{j+1}} |a_n f_n| \right\|_p \sim \left\| \sum_{n=2^j+1}^{2^{j+1}} a_n f_n \right\|_p \sim 2^{j(1/2-1/p)} \left(\sum_{n=2^j+1}^{2^{j+1}} |a_n|^p \right)^{1/p}$$

   for arbitrary real a_n *with the constants in the equivalence relation* $\sim$ *independent of* p *and* j. (*Properties of the orthonormal Franklin system, II*, Studia Math. 27, 289–323)
2. (Przemysław Wojtaszczyk, 1982) Theorem:
   The Franklin system is an unconditional basis in Hardy's space $H^1(I)$. (*The Franklin system is an unconditional basis in* H_1, Ark. Mat. 20, 293–300)
3. (Zbigniew Ciesielski and Tadeusz Figiel, 1983) Construction of bases with optimal properties for important classes of function spaces on an arbitrary smooth compact manifold. (Z. Ciesielski and T. Figiel, *Spline bases in classical function spaces on compact* C^∞ *manifolds*, Studia Math. 76, Part II, 95–136)

Operational Calculi

(former Department of Mathematical Analysis, former Department of Generalized Functions)

Results:

1. (Jan Mikusiński, 1950–52) Creation of Mikusiński's operational calculus (*Rachunek Operatorowy*, PWN 1953).
2. (Danuta Przeworska-Rolewicz, 1973–1988) Creation of a theory called algebraic analysis (*Algebraic Analysis*, PWN & D. Reidel 1988).

Monographs:

1. D. Przeworska-Rolewicz, *Equations with transformed arguments. An algebraic approach*, Elsevier 1973.
2. J. Mikusiński, *The Bochner Integral*, Academic Press 1978, Birkhäuser 1987
3. J. Mikusiński, *Operational Calculus*, vol. I, Pergamon Press 1983.
4. J. Mikusiński & T. K. Boehme, *Operational Calculus*, vol. II, Pergamon Press 1987.
5. P. Antosik & C. Swartz, *Matrix Methods in Analysis*, Springer 1985.
6. D. Przeworska-Rolewicz, *Logarithms and antilogarithms. An algebraic approach*, with appendix by Z. Binderman, Kluwer 1998.
7. K. Skórnik (with H.-J. Glaeske, A. P. Prudnikov), *Operational Calculus and Related Topics*, Chapman & Hall/CRC 2006.

Functional Analysis

(Department of Functional Analysis,

former Department of Mathematical Analysis, Department of Probability Theory)

Results:

1. (Wiesław Żelazko with J. P. Kahane, 1968) Theorem:
   A linear functional F (not necessarily continuous) on a complex Banach algebra A with unity e is multiplicative, if and only if $F(e) = 1$ and $F(x) \neq 0$ for all invertible elements x of A. (J. P. Kahane & W. Żelazko, *A characterization of maximal ideals in commutative Banach algebras*, Studia Math. 29, 339–343; W. Żelazko, *A characterization of multiplicative linear functionals in complex Banach algebras*, Studia Math. 30, 83–85)
2. (Aleksander Pełczyński, 1969) Theorem:
   There exists a separable Banach space B with a basis such that every separable Banach space with a basis is isomorphic to a complemented subspace of B. (*Universal bases*, Studia Math. 32, 247–268)

Definition 1. If K is a unit ball in a Banach space X and $d \geq 1$, then we say that a linear subspace $Y \subseteq X$ is d-hilbertian, if there exists an ellipsoid $E \subset Y$ such that $E = -E$ and $K \cap Y \subseteq E \subseteq dK$.

Definition 2. Metric spaces X and Y are Lipschitz equivalent, if there exists a homomorphizm f of X onto Y such that f and f^{-1} satisfy the Lipschitz condition.

Results: (continuation)

3. (Tadeusz Figiel with J. Lindenstrauss and V. Milman, 1977) Theorem:
For every $\tau > 0$ *there is a constant* $\eta(\tau) > 0$ *having the following property. Let* $(X, || \ ||)$ *be a* n*-dimensional Banach space and let* $||| \cdot |||$ *be an inner product norm on* X *such that* $a|||x||| \leq ||x|| \leq b|||x|||$ *for suitable positive* a, b*. Let* M_r *be the median of* $r(x) = ||x||$ *on* $\{x, |||x||| = 1\}$*. Then for* $k = [\eta(\tau) n M_r^2 / b^2]$ *there is a* k*-dimensional subspace* Y *of* X *which is* $(1 + \tau)$*-hilbertian.* (*The dimension of almost spherical sections of convex bodies*, Acta Math. 139, 53–94)
4. (Piotr Mankiewicz with S. Heinrich, 1982) Theorem:
If E *is a Banach space Lipschitz equivalent to the space* l_p *of scalar sequences, whose* p*-th powers are absolutely summable or to the space* $L_p(0,1)$ *of scalar functions whose* p*-th powers are absolutely Lebesgue integrable for a certain* $p > 1$*, then* E *is isomorphic to* l_p *or* $L_p(0,1)$*, respectively.* (*Applications of ultrapowers to the uniform and Lipschitz classification of Banach spaces*, Studia Math. 73, 225–251)

Monographs:

1. Z. Semadeni, *Banach spaces of continuous functions*, vol. 1, PWN 1971.
2. S. Rolewicz, *Metric linear spaces*, PWN 1972, 2nd ed. 1985.
3. P. Wojtaszczyk, *Banach spaces for analysts*, Cambridge Univ. Press 1991.

Theory of Distributions

(former Department of Mathematical Analysis, former Department of Generalized Functions)

Results:

1. (Jan Mikusiński and Roman Sikorski, 1957) Creation of sequential theory of distributions (*The elementary theory of distributions I*, Dissert. Math. 12, 1–54).
2. (Stanisław Łojasiewicz, 1959) Theorem about dividing a distribution by an analytic function: *An equation* $\phi S = T$ *has a solution* S *for every distribution* T *and every analytic real function* $\phi \not\equiv 0$. (*Sur le problème de la division*, Studia Math. 18, 87–136)

Monographs:

1. J. Mikusiński & R. Sikorski, *Théorie élémentaire des distributions*, Gauthier-Villars 1964.
2. P. Antosik, J. Mikusiński, R. Sikorski, *Theory of distributions – the sequential approach*, Elsevier 1973.

Operator Theory

(former Department of Real Functions, former Department of Mathematical Analysis, Laboratory of Hilbert Space, Laboratory of Numerical Analysis)

Results:

1. (Włodzimierz Mlak, 1963) Logarithm theorem:
   The spectral measure of minimal unitary dilatation of a contraction (without unitary part) is absolutely continuous with respect to the Lebesgue measure, moreover Radon–Nikodym derivative of this measure is logarithmically integrable on the unit circle with respect to the Lebesgue measure on this circle. (*Characterization of completely non-unitary contractions in Hilbert space*, Bull. Acad. Polon. Sci. 11, 111–113)
2. (Czesław Ryll-Nardzewski, 1966) Fixed point theorem:
   Let S denote a semigroup of linear operators acting in Banach space B with norm $||\cdot||$, preserving a weakly compact and convex set C. If for any pair of points $x, y \in B$, $x \neq y$, $\inf_{s\in S}||sx - sy|| > 0$, *then there exists $x_0 \in C$ such that for any $s \in S : sx_0 = x_0$.* (*On fixed points of semi-groups of endomorphisms of linear spaces*, Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability, vol. II, part 1, 55–61)

3. (Stefan Rolewicz, 1969) Theorem about existence of hypercyclic operators:
Let X be a Banach space. Let T be a continuous linear operator acting in X. We say that T is hypercyclic if there is an element $x \in X$, such that its orbit, i.e. the set $\{x, Tx, T^2x, ...\}$, is dense in X. It was shown that in the space l^p there are hypercyclic operators. (*On orbits of elements*, Studia Math. 32, 18–22)
The paper was a starting point of investigations of hypercyclic operators. Now there is a section devoted to this subject in Mathematical Reviews (section 47A16).
4. (Andrzej Pokrzywa, 1979) Theorem:
Any part of the spectrum of a bounded linear operator A acting in a Hilbert space which is disjoint with its essential numerical range is approximated by the respective parts of spectra of operators A_n defined by

$$A_n := P_n A|_{\operatorname{ran} P_n},$$

where $\{P_n\}$ is a sequence of orthogonal projections converging strongly to I. (*Method of orthogonal projections and approximation of the spectrum of a bounded operator*, Part I, Studia Math. 65, 21–29, Part II ibid. 70, 1–9)

5. (Andrzej Pokrzywa, 1980) Theorem:

If A is a compact perturbation of a selfadjoint operator by an operator belonging to the Macaev ideal, then the spectra of operators A_n asymptotically cover the spectrum of A. Moreover all the spectral invariant spaces of A are approximated by the respective spectral subspaces of operators A_n. (*Spectra of compressions of an operator with compact imaginary part*, J. Operator Theory 3, 151–158; *Limits of spectra of strongly converging compressions*, ibid. 12, 199–212)

Control Theory

(Department of Differential Equations, former Department of Mathematical Analysis)

Result:

1. (Czesław Olech, 1989) Theorem:

   Let (Ω, Σ, μ) *be a measurable space:* Ω *is an abstract space,* Σ *is* σ*-field of subsets of* Ω *and* $\mu : \Sigma \to \mathbb{R}^n$ *a non atomic finite measure. Let* $K \subset L^1(\Omega, \mathbb{R}^n)$ *be a closed and decomposable set of vector functions, decomposable means that for any two functions* f *and* g *from* K *and any measurable set* A *the function* $f\chi_A + g\chi_{\Omega\setminus A}$ *is also in* K *(*χ *is the characteristic function of a set).*

   Then the set of integrals of functions from K

   $$I(K) = \left\{ \int_\Omega f|\mu| : f \in K \right\}$$

   is convex in $\mathbb{R}^n$*. If* $I(K)$ *is bounded, then it is compact and each point* x *of this set is a finite sum of extreme elements* w_i *of family* K

   $$x = \sum_i \int_{A_i} w_i|\mu|,$$

   where $i = 1, \ldots, k$ *and* $k \le n+1$*, the sets* $A_1, \ldots, A_{k+1}$ *form a measurable disjoint cover of* Ω *and belong to a certain extreme independent from* x *subfield of* Σ*.* (*The Lyapunov theorem, its extensions and applications*, LNM 1446, 84–103, Springer)

Monograph:

D. Pallaschke & S. Rolewicz, *Foundations of mathematical optimization. Convex analysis without linearity*, Kluwer 1997.

Differential Geometry

(former Department of Complex Analysis and Differential Geometry,
Department of Differential Geometry and Mathematical Physics)

Results:

1. (Stanisław Gołąb and Halina Pidek-Łopuszańska, 1957) Making clear the notion of algebra of geometric objects. (*Sur l'algèbre des objects géométriques de première classe à une composante*, Ann. Polon. Math. 4, 226–241)

2. (Janusz Grabowski and Katarzyna Grabowska, 2008) On a vector bundle equipped with a structure of a general algebroid, a concept of variational calculus with constraints is developed and the corresponding Euler-Lagrange equations with constraints of different types are derived. They reduce to the standard ones for the case of tangent bundles. This framework unifies in a geometrical way most of first order Lagrangian systems known in the literature. (*Variational calculus with constraints on general algebroids*, J. Phys. A: Math. Theor. 41, 175204, 25 pp.)

Monograph:

W. Ślebodziński, *Formes extérieures et leurs applications*, vol. I PWN 1954, vol. II PWN 1963.

Topology (Department of Topology)

Results:

1. (Karol Borsuk, 1948–1967) Creation of the Theory of Retracts (*Theory of retracts*, PWN1967).
2. (Karol Borsuk, 1968–1975) Creation of the Theory of Shape (*Theory of shape*, PWN 1975).
3. (Henryk Toruńczyk, 1977,78) Characterization of manifolds modelled on the Hilbert cube or on the Hilbert space. (*On CE-images of the Hilbert cube and characterization of Q-manifolds*, Fund. Math. 106, 30–41; *Characterizing Hilbert space topology*, ibid. 111, 247–262)

Monographs:

1. K. Kuratowski, *Topology*, vol. 1, Academic Press 1966, vol. 2, Academic Press 1968.
2. K. Borsuk, *Theory of retracts.*, Monografie Matematyczne, Tom 44. [Mathematical Monographs, Vol. 44 PWN—Polish Scientific Publishers, Warsaw, 1975.]
3. K. Borsuk, *Theory of shape.*, Monografie Matematyczne, Tom 59. [Mathematical Monographs, Vol. 59 PWN—Polish Scientific Publishers, Warsaw, 1975. 379 pp.]
4. C. Bessaga & A. Pełczyński, *Selected topics in infinite dimensional topology*, PWN 1975.

Noncommutative Geometry

(Laboratory of Noncommutative Geometry)

Hopf-cyclic cohomology was discovered by Alain Connes and Henri Moscovici in their seminal series of papers on the characteristic classes for the transverse geometry of foliations. Their construction of a cyclic complex hinged on a very lengthy computation whose final result looked like a miracle. The article "Hopf-cyclic homology and cohomology with coefficients" by Piotr M. Hajac, Masoud Khalkhali, Bahram Rangipour and Yorck Sommerhaeuser generalized this complex in a way that afforded not only a one-line conceptual proof of the cyclicity of the complex, but also introduced the much desired coefficients into cyclic theory. This work opened a door to new structures that are quite intensively explored, and was honoured by IMPAN as one of the seven best papers written by IMPAN's employees in 2003.

Probability Theory

(Department of Probability Theory and Mathematical Finance)

Results:

1. (Tomasz Komorowski and S. Olla, 2001) Proof that a motion of a particle in incompressible velocity field satisfies a central limit theorem. (*On homogenization of time dependent random flows*, Probab. Theory Related Fields 121, no. 1, 98–116)
2. (Szymon Peszat and Jerzy Zabczyk, 2000) Proof of existence of a solution to a stochastic wave equation with spatially homogeneous noise in three dimensions. (*Nonlinear stochastic wave and heat equations*, Probab. Theory Related Fields 116, 421–443)
3. (Łukasz Stettner and G. Di Masi, 1999) Proof of existence of a solution to Bellman equation for control problems with discrete time and risk-sensitive cost functional. (*Risk-sensitive control of discrete-time Markov processes with infinite horizon*, SIAM J. Control Optim. 38, no. 1, 61–78)

Monographs:

1. G. Da Prato and J. Zabczyk, *Second Order Partial Differential Equations in Hilbert Spaces*, Cambridge University Press, 2002.
2. S. Peszat and J. Zabczyk, *Stochastic Partial Differential Equations with Lévy noise*, Cambridge University Press, 2007.

Statistics (former Department of Mathematical Statistics and its Applications)

Results:

1. (Kazimierz Florek, Józef Łukaszewicz, Julian Perkal, Hugo Steinhaus, Stefan Zubrzycki, 1951) Introduction of the so called Wrocław taxonomy. (*Sur la liaison et la division des points d'un ensemble fini*, Colloq. Math. 2, 282–285)
2. (Tadeusz Bednarski, 1993) Solution to the problem of robust estimation in the Cox regression model. (*Robust estimation in the Cox regression model model*, Scand. J. Statist. 20, 213–225)
3. (Tadeusz Inglot, W. Kallenberg, Teresa Ledwina, 1998) Proof of the result on vanishing of the shortcoming of data driven Neyman's tests. (*Vanishing shortcoming of data driven Neyman's tests*, Asymptotic Methods in Probability and Statistics, A Volume in Honour of Miklós Csörgő. B. Szyszkowicz (ed.), Elsevier, Amsterdam, 811–829)