Uniqueness results for classes of semipositone problems

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Abstract

We consider steady state reaction diffusion equations on the exterior of a ball, namely, boundary value problems of the form:

$$\begin{cases} -\Delta_p u = \lambda K(|x|) f(u) & \text{in } \Omega_E, \\ u = 0 & \text{on } |x| = r_0, \\ u \to 0 & \text{when } |x| \to \infty, \end{cases}$$

where $\Delta_p z := \text{div}(|\nabla z|^{p-2}\nabla z)$, $1 < p < n$, $\lambda > 0$ and $\Omega_E := \{x \in \mathbb{R}^n \mid |x| > r_0 > 0\}$. Here the weight function $K \in C^1([r_0, \infty), (0, \infty))$ satisfies $\lim_{r\to\infty} K(r) = 0$, and the reaction term $f \in C^1([0, \infty))$ is strictly increasing and satisfies $f(0) < 0$, $\lim_{s\to\infty} f(s) = \infty$, $\lim_{s\to\infty} \frac{f(s)}{s^{p-1}} = 0$ and $\frac{f(s)}{s^q}$ is nonincreasing on $[a, \infty)$ for some $a > 0$ and $q \in (0, p-1)$. We establish uniqueness results for positive radial solutions for $\lambda \gg 1$.