Dynamic Games in Biomedical Problems

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The paper outlines our experience in application of dynamic differential games to some biomedical problems.

1. Krasovskii's unification method [1] is used to effectively solve a Hamilton-Jacobi, eikonal, equation describing the propagation of acoustic wave fronts in the so-called HPSW (Horizontally Polarized Shear Wave) biosensor. Such a sensor serves for the detection and quantitative measurement of microscopic amounts of biological substances. The wave fronts can be found very precisely even in the case of very complicated geometry of the wave excitation source.

2. Numerical methods [2] developed by the authors for solving Hamilton-Jacobi equations are utilized to optimize cooling and thawing protocols in cryopreservation of living cells and tissues. The objective of modeling is to reduce cell damage occurring because of sudden supercooling temperature drops, shrinkage of cells caused by the osmotic flow through cell membranes, or the development of dendrite seeds. The results are applied to a real freezing plant.

3. Based on viability approach [3, 4], a mathematical model of impaired cerebral autoregulation (a passive, linear, dependency of the cerebral blood flow on the arterial mean pressure) is proposed and feedback controls that are able to compensate failures of autoregulation are constructed. Impaired cerebral autoregulation can lead to damage of fragile blood vessels of the germinal matrix being a highly vascularized layer of neuronal and glial precursors in premature brain. As a consequence, a brain bleeding and a subsequent lifelong disability can occur. This investigation is a joint project with the Research Unit of the Buhl-Strohmaier Foundation for Pediatric Neuroorthopaedics and Cerebral Palsy (Clinic 'rechts der Isar' of the Technical University of Munich) headed by Prof. Dr. med. R. Lampe.

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