Discrete-Time Dynamic Potential Games

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We consider the dynamic potential games. The main goal is to construct the potential in case of the Nash equilibrium among feedback strategies. The class of games where such potential exists is obtained. As an illustration the discrete-time harvesting problem is investigated. The potential is obtained in the case of identical and asymmetric players.

As an illustration of this games we consider the discrete-time game-theoretic models related to resource management problem (common resource exploitation) are investigated. The players (countries or firms) which harvest the common stock on infinite time horizon are the participants of the game. We construct Nash equilibrium in feedback strategies.

Let two players exploit the common resource. The state dynamics is in the form

$$x_{t+1} = f(x_t, u_{1t}, u_{2t}) \,, \;\; x_0 = x \,, \tag{1}$$

where $x_t \geq 0$ is the size of the resource at a time t, $f(x_t, u_{1t}, u_{2t})$ means natural growth function, $u_{it} \geq 0$ gives the strategy (exploitation intensity) of player i, $i = 1, 2$.

The payoff functions of the players over infinite time horizon are defined by

$$J_i = \sum_{t=0}^{\infty} \delta^t g_i(u_{1t}, u_{2t}) \,, \;\; i = 1, 2 \,, \tag{2}$$

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where $0 < \delta < 1$ means the common discount factor, $g_i(u_{1t}, u_{2t}) \geq 0$ gives the instantaneous utility, $i = 1, 2$.

The main question arising here is how to construct the potential in order to solve single multivariate optimal control problem instead of a set of coupled optimal control problems [1–4]. The optimal strategies are obtained in the feedback form $u_{it} = u_i(x_t)$, $i = 1, 2$, so the classical approaches [1,4] can't be applied. Here we consider only the case of linear growth function and quadratic instantaneous utilities. We determine the potential in quadratic form and obtain the restriction to the class of games where such potential exists.

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