OPTIMAL UNIFORM APPROXIMATION OF LÉVY PROCESSES ON BANACH SPACES WITH FINITE VARIATION PROCESSES

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Let X_t, $t \geq 0$, be a cádlag Lévy Process on a Banach space V (i.e. a process with a.s. cadlag paths and independent and stationary increments), and let $\mathcal{A}_X$ be the family of V-valued processes Y_t, $t \geq 0$, adapted to the natural filtration of X. By $|\cdot|$ we denote the norm in V. For $T > 0$ and two processes $T, Z \colon \Omega \times \mathcal{T} \to B$, where $\mathcal{T}$ is an index set such that $[0, T] \subset \mathcal{T}$, we denote

$$\|Y - Z\|_{\infty,[0,T]} := \sup_{0 \leq t \leq T} |Y_t - Z_t|$$

and

$$\mathrm{TV}(Y, [0, T]) := \sup_n \sup_{0 \leq t_0 < t_1 < \ldots < t_n \leq T} \sum_{i=0}^{n} |Y_{t_i} - T_{t_{i-1}}|.$$

In this talk we will deal with the following optimisation problem. For givennon-decreasing function $\phi \colon [0, +\infty) \to [0, +\infty)$ and $T, \theta > 0$, calculate (or at least estimate up to universal constants)

$$V_X(\psi, \theta, T) := \mathbb{E}\left\{\psi\left(\|X - Y\|\right) + \theta \cdot \mathrm{TV}(Y, [0, T])\right\}. \tag{1}$$

To make the problem non-trivial we assume that $\mathbb{E}|X_1| < +\infty$.

Such problems arise for example in financial mathematics when the process X denotes optimal hedging strategy while Y denotes approximation of this startegy in the presence of (proportional) transaction costs.

We will present formulas for $V_X(\psi, \theta, T)$ expressed in terms of simpler func-tionals, like $\mathbb{E}|X_{\tau^c}|\mathbf{1}_{\{\tau^c < T\}}$ and $\mathbb{E}\exp\left(-\tau^c/T\right)$, where

$$\tau^c := \inf\{t > 0 \colon |X_t| > c\}.$$

We will present also more explicit solutions in special cases when X is a Brownian motion with drift on $\mathbb{R}$, standard Brownian motion on $\mathbb{R}^n$ or symmetric α-stable process on $\mathbb{R}$.