The continuous field of quantum $GL(N, \mathbb{C})$

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Overview

Main goal

Quantum $GL(N, \mathbb{C})$, $U(N)$, $T(N)$ and $PD(N)$

s*-algebras

$\mathscr{R}$-algebras and fields of C*-algebras

Main goal

Goal

Consider Hopf $\mathbb{C}[\mathbf{q}, \mathbf{q}^{-1}]$*-algebra*

$$(\mathcal{O}_{\mathbf{q}}(GL(N, \mathbb{C})), \Delta).$$

Find 'completion'

$$(C_{0,\mathbf{q}}(GL(N, \mathbb{C})), \Delta)$$

as continuous field of Hopf C^**-algebras over* $(0, +\infty)$*.*

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$\rightsquigarrow$ Focus today: $C_{0,\mathbf{q}}(GL(N,\mathbb{C}))$ as continuous field of C*-algebras.

$\rightsquigarrow$ For quantum $U(N)$: done by G. Nagy ('00).

Quantum $GL(N,\mathbb{C})$, $U(N)$, $T(N)$ and $PD(N)$

Unital $*$-algebras and closed spectral conditions

Definition

Unital $*$-algebra $\mathscr{A}$.

Closed spectral conditions: family $\mathscr{S}$ of statements

'Joint spectrum of the commuting normal elements

$y_1, \ldots, y_m \in M_N(\mathscr{A})$ lies in closed set $S \subseteq \mathbb{C}^m$.'

$\rightsquigarrow$ Consider bounded representations preserving spectral conditions.

$\rightsquigarrow$ Notion of positive elements.

Quantum $GL(N, \mathbb{C})$, $U(N)$, $T(N)$ and $PD(N)$

R-matrix

Write $\mathbb{C}_\mathbf{q} = \mathbb{C}[\mathbf{q}, \mathbf{q}^{-1}]$ as $\mathbb{C}$-algebra, $\mathbf{q}^* = \mathbf{q}$, $\mathbf{q} \geq 0$.

Definition

Quantum R-matrix associated to $GL(N, \mathbb{C})$:

$$\mathbf{R} \in M_N(\mathbb{C}_\mathbf{q}) \otimes M_N(\mathbb{C}_\mathbf{q}),$$

$$\mathbf{R} = \mathbf{q}^{-1} \sum_i e_{ii} \otimes e_{ii} + \sum_{i \neq j} e_{ii} \otimes e_{jj} + (\mathbf{q}^{-1} - \mathbf{q}) \sum_{i<j} e_{ij} \otimes e_{ji}$$

Note: $\mathbf{R}^* = \mathbf{R}_{21}$ and

$$\mathbf{R}_{12}\mathbf{R}_{13}\mathbf{R}_{23} = \mathbf{R}_{23}\mathbf{R}_{13}\mathbf{R}_{12}.$$

Complex quantum $GL(N,\mathbb{C})$

Definition

Complex quantum $M_N(\mathbb{C})$: $\mathbb{C}_{\mathbf{q}}$-bialgebra $\mathcal{O}_{\mathbf{q}}(M_N(\mathbb{C}))$ with:

- Generators X_{ij} as $\mathbb{C}_{\mathbf{q}}$-algebra,

$$X = (X_{ij})_{ij} \in M_N(\mathcal{O}_{\mathbf{q}}(GL(N,\mathbb{C}))) = M_N(\mathbb{C}_{\mathbf{q}}) \underset{\mathbb{C}_{\mathbf{q}}}{\otimes} \mathcal{O}_{\mathbf{q}}(GL(N,\mathbb{C})).$$

- Relations

$$\mathbf{R}_{12}X_{13}X_{23} = X_{23}X_{13}\mathbf{R}_{12},$$

- Coproduct

$$(\mathrm{id}\otimes\Delta)(X) = X_{12}X_{13}.$$

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Complex quantum $GL(N,\mathbb{C})$: Hopf $\mathbb{C}_\mathbf{q}$-algebra

$$\mathcal{O}_\mathbf{q}(GL(N,\mathbb{C})) = \mathcal{O}_\mathbf{q}(M_N(\mathbb{C}))[\mathrm{Det}_\mathbf{q}(X)^{-1}].$$

Quantum $GL(N,\mathbb{C})$, $U(N)$, $T(N)$ and $PD(N)$

Real quantum $GL(N,\mathbb{C})$

Definition

Real quantum $GL(N,\mathbb{C})$: Hopf $\mathbb{C}_{\mathbf{q}}$-$*$-algebra $\mathcal{O}_{\mathbf{q}}^{\mathbb{R}}(GL(N,\mathbb{C}))$ with:

- Quantum generators $X, Y \in M_N(\mathcal{O}_{\mathbf{q}}^{\mathbb{R}}(GL(N,\mathbb{C}))$ over $\mathbb{C}_{\mathbf{q}}$.
- Relations

$$\mathbf{R}_{12}X_{13}X_{23} = X_{23}X_{13}\mathbf{R}_{12}, \qquad (\mathrm{id}\otimes\Delta)(X) = X_{12}X_{13}.$$

$$\mathbf{R}_{12}Y_{13}Y_{23} = Y_{23}Y_{13}\mathbf{R}_{12}, \qquad (\mathrm{id}\otimes\Delta)(Y) = Y_{12}Y_{13}.$$

$$\mathbf{R}_{12}X_{13}Y_{23} = Y_{23}X_{13}\mathbf{R}_{12}, \qquad X^*Y = YX^* = I_N.$$

Quantum $U(N)$ and quantum $T(N)$

Definition

- Quantum unitary group $\pi_U : \mathcal{O}_{\mathbf{q}}(GL(N,\mathbb{C})) \twoheadrightarrow \mathcal{O}_{\mathbf{q}}(U(N))$,

$$X \mapsto U, \quad Y \mapsto U.$$

- Quantum triangular group $\pi_T : \mathcal{O}_{\mathbf{q}}(GL(N,\mathbb{C})) \twoheadrightarrow \mathcal{O}_{\mathbf{q}}(T(N))$,

$$X \mapsto T, \quad Y \mapsto (T^*)^{-1}$$

where

$$T_{ij} = 0 \text{ for } i > j, \qquad T_{ii}^* = T_{ii}, \qquad T_{ii} \geq 0.$$

Quantum $H(N)$ and quantum $PD(N)$

Definition

- Quantum Hermitian matrices $\mathcal{O}_{\mathbf{q}}(H(N))$: $\mathbb{C}_{\mathbf{q}}$-algebra with
  - quantum generator $Z \in M_N(\mathcal{O}_{\mathbf{q}}(H(N))$ with $Z^* = Z$,
  - Z satisfies Reflection Equation

$$\mathbf{R}_{21}Z_{13}\mathbf{R}_{12}Z_{23} = Z_{23}\mathbf{R}_{21}Z_{13}\mathbf{R}_{12}.$$

- Quantum positive-semidefinite matrices $\mathcal{O}_{\mathbf{q}}(P(N))$:

$$\text{Spectral condition } Z \geq 0.$$

- Quantum positive-definite matrices $\mathcal{O}_{\mathbf{q}}(PD(N))$:

$$\mathcal{O}_{\mathbf{q}}(PD(N)) = \mathcal{O}_{\mathbf{q}}(P(N))[\mathrm{Det}_{\mathbf{q}}(Z)^{-1}].$$

Quantum $GL(N,\mathbb{C})$, $U(N)$, $T(N)$ and $PD(N)$

Quantum analogues of classical decompositions

Proposition (DC-Floré ('17))

Quantum QR-decomposition

$$\chi^{\mathbf{q}}_{UT} = (\pi_U \otimes \pi_T)\Delta : \mathcal{O}^{\mathbb{R}}_{\mathbf{q}}(GL(N,\mathbb{C})) \hookrightarrow \mathcal{O}_{\mathbf{q}}(U(N)) \underset{\mathbb{C}_{\mathbf{q}}}{\otimes} \mathcal{O}_{\mathbf{q}}(T(N)).$$

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Proposition (Joseph-Letzter ('92))

Quantum Cholesky map

$$\chi^{\mathbf{q}}_{T} : \mathcal{O}_{\mathbf{q}}(PD(N)) \hookrightarrow \mathcal{O}_{\mathbf{q}}(T(N)), \qquad Z \mapsto T^*T.$$

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$$\chi_T^{\mathbf{q}} : \mathcal{O}_{\mathbf{q}}(PD(N)) \hookrightarrow \mathcal{O}_{\mathbf{q}}(T(N)), \qquad Z \mapsto T^*T.$$

Proposition

Unitary conjugation of positive-definite matrices

$$\mathrm{Ad}_{\mathbf{q}} : \mathcal{O}_{\mathbf{q}}(PD(N)) \to \mathcal{O}_{\mathbf{q}}(PD(N)) \underset{\mathbb{C}_{\mathbf{q}}}{\otimes} \mathcal{O}_{\mathbf{q}}(U(N)), \quad Z \mapsto U_{13}^* Z_{12} U_{13}.$$

Quantum $GL(N, \mathbb{C})$, $U(N)$, $T(N)$ and $PD(N)$

Cayley-Hamilton

Definition

Let $\mathscr{A}$ a $*$-algebra.

$V \in M_N(\mathscr{A})$ is Cayley-Hamilton with leading Casimir $C \in \mathscr{Z}(\mathscr{A})$:

$$V^N + CV^{N-1} + C_2V^{N-2} + \ldots + C_{N-1}V + C_N = 0$$

for $C, C_k \in \mathscr{Z}(\mathscr{A})$.

Proposition (Pyatov-Saponov ('95); Jordan-White ('17), DC-Floré ('17))

The quantum matrices $V = Z, X^*X, T^*T$ are Cayley-Hamilton,

$$-C = \mathrm{Tr}_{\mathbf{Q}}(V) = \sum_i \mathbf{q}^{2N-2i} V_{ii}.$$

Quantum $GL(N,\mathbb{C})$, $U(N)$, $T(N)$ and $PD(N)$

Conditional expectation

Extend $\mathbb{C}_{\mathbf{q}}$ to $k = \mathbb{C}_{\mathbf{q}}[n_{\mathbf{q}}^{-1}]$ where $n_{\mathbf{q}} = \frac{\mathbf{q}^n - \mathbf{q}^{-n}}{\mathbf{q} - \mathbf{q}^{-1}}$.

Proposition (Gerstenhaber-Schaps ('97))

There exists an invariant normalized functional $\Phi : \mathcal{O}_{\mathbf{q}}(U(N)) \to k$.

Proposition

$$E : \mathcal{O}_{\mathbf{q}}(PD(N)) \to \mathcal{O}_{\mathbf{q}}(PD(N))^{\mathrm{Ad}_{\mathbf{q}}} \subseteq \mathscr{Z}(\mathcal{O}_{\mathbf{q}}(PD(N)),$$

$$E(a) = (\mathrm{id} \otimes \Phi)\,\mathrm{Ad}_{\mathbf{q}}(a).$$

$*$-algebras and filtration of Irreps

Definition

Let

- $\mathscr{A}$ unital $*$-algebra with closed spectral conditions,
- P max. collection inequivalent irred. bounded reps,
- $\mathscr{P} = \{P_{\leq M}\}$ positive filtration of P,
- For any bounded π, $P_\pi = \{\pi' \in P \mid \pi' \preceq \pi\}$.

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- For any bounded π, $P_\pi = \{\pi' \in P \mid \pi' \preceq \pi\}$.

We call $\mathscr{P}$ a C*-filtration if

- $\forall a \in \mathscr{A}$ and M, $\sup_{\pi \in P_{\leq M}} \|\pi(a)\| < \infty$.
- For each bounded π there exists M with $P_\pi \subseteq P_{\leq M}$.

$\rightsquigarrow$ Notion of equivalent C*-filtrations $\mathscr{P} \sim \mathscr{P}'$.

Example

Example

1. - $\mathscr{A}$ unital $*$-algebra
   - $X \in M_N(\mathscr{A})$
   - $P^X_{\leq M} = \{\pi \in P \mid \|\pi(X)\| \leq M\}$,
   - $\mathscr{P}^X$ C*-filtration if $\mathscr{A} = \mathbb{C}\langle X_{ij}, X_{ij}^* \rangle$ (X is quantum generator).

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2.
   - $\mathscr{A} = \mathbb{C}[\mathbf{q}]$, $\mathbf{q}^* = \mathbf{q}$,
   - $P \cong \mathbb{R}$,
   - $P_{\leq M} = [-M, M]$.

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   - $P \cong \mathbb{R}$,
   - $P_{\leq M} = [-M, M]$.
3.
   - $\mathscr{A} = \mathbb{C}_{\mathbf{q}} = \mathbb{C}[\mathbf{q}, \mathbf{q}^{-1}]$, $\mathbf{q}^* = \mathbf{q}$, closed spectral condition $\mathbf{q} \geq 0$.
   - $P \cong (0, +\infty)$,
   - $P_{\leq M} = [M^{-1}, M]$.

Definition

Let $\mathscr{A}$ unital $*$-algebra with C*-filtration $\mathscr{P}$.

- $A_{\leq M}$ separation-completion $\mathscr{A}$ for $\|a\|_M = \sup_{\pi \in P_{\leq M}} \|\pi(a)\|$.
- $A_b = \varprojlim^b A_{\leq M} \subseteq \prod^b_{\pi \in P} B(\mathcal{H}_\pi)$.
- $P_{\geq M} = P \setminus \cup_{M' < M} P_{\leq M'}$,
- $A_{<M} = \{a \in A_b \mid \pi(a) = 0 \text{ for } \pi \in P_{\geq M}\} \subseteq A_b$,
- $A_c = \cup_M A_{<M} \subseteq A_b$,
- $A_0 = \text{closure } A_c$.

We call A_0 the C*-hull of $\mathscr{A}$ with respect to $\mathscr{P}$.

s*-algebras

Example

Let

- $\mathscr{A} = \mathbb{C}_{\mathbf{q}}$, $\mathbf{q}^* = \mathbf{q}$, closed spectral condition $\mathbf{q} \geq 0$.
- $P \cong (0, +\infty)$,
- $P_{\leq M} = [M^{-1}, M]$.

Then

- $A_{\leq M} = C_b([M^{-1}, M])$,
- $A_b = C_b((0, +\infty))$,
- $A_{<M} = C_0((M^{-1}, M))$,
- $A_c = C_c((0, +\infty))$,
- $A_0 = C_0((0, +\infty))$.

$\rightsquigarrow$ Same works for $k = \mathbb{C}_{\mathbf{q}}[n_{\mathbf{q}}^{-1}]$.

Questions and example

Questions

In general:

- *When is* $A_b = M(A_0)$*?*
- *When is* $\mathrm{Irrep}(\mathscr{A}) \cong \mathrm{Irrep}(A_0)$*?*

Compact Tietze extension property

Definition

We say $(\mathscr{A}, \mathscr{P})$ satisfies compact Tietze extension property if

$$\text{For all } M \text{ there exists } M' > 0 \text{ such that } A_{<M'} \twoheadrightarrow A_{\leq M}.$$

Theorem

If $(\mathscr{A}, \mathscr{P})$ satisfies the Tietze extension theorem, then

- $A_b = M(A_0)$ *and*
- $\mathrm{Irrep}(\mathscr{A}) \cong \mathrm{Irrep}(A_0)$.

s*-algebras

s*-algebras

Definition

Quantum control: $X \in M_N(\mathscr{A})$ with $\mathscr{P}^X$ C*-filtration.

Quantum control for $\mathscr{P}$: $\mathscr{P}^X \sim \mathscr{P}$

Central control: quantum control $\oplus_k^N C_k$ with $C_k \in \mathscr{Z}(\mathscr{A})$.

Note: X quantum control $\iff$ X^*X quantum control.

s*-algebras

s*-algebras

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Definition

We call $(\mathscr{A}, \mathscr{P})$ a weak s*-algebra if $\exists$ quantum control for $\mathscr{P}$.

We call $(\mathscr{A}, \mathscr{P})$ a s*-algebra if $\exists$ central control for $\mathscr{P}$.

s*-algebras

s*-algebras

Definition

Quantum control: $X \in M_N(\mathscr{A})$ with $\mathscr{P}^X$ C*-filtration.

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Definition

We call $(\mathscr{A}, \mathscr{P})$ a weak s*-algebra if $\exists$ quantum control for $\mathscr{P}$.

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Theorem

Any s-algebra has the Tietze extension property.*

Criterion for s*-algebras

Definition

$\mathscr{A}$ a unital $*$-algebra and $X \in M_N(\mathscr{A})$. For $\mathbf{q}_1, \ldots, \mathbf{q}_N \in \mathscr{Z}(\mathscr{A})$,

$$\mathrm{Tr}_{\mathbf{Q}}(X) = \sum_i \mathbf{q}_i X_{ii}, \qquad \mathbf{Q} = \begin{pmatrix} \mathbf{q}_1 & & & \\ & \mathbf{q}_2 & & \\ & & \ddots & \\ & & & \mathbf{q}_N \end{pmatrix}.$$

Theorem

Let $(\mathscr{A}, \mathscr{P})$ *weak s*-algebra with positive quantum control* $\mathbf{Q} \oplus X$.

Assume $\mathrm{Tr}_{\mathbf{Q}}(X) \in \mathscr{Z}(\mathscr{A})$.

Then $(\mathscr{A}, \mathscr{P})$ *s*-algebra with positive central control* $\mathbf{Q} \oplus \mathrm{Tr}_{\mathbf{Q}}(X)$.

s*-algebras

More examples

Proposition

The k-algebra $\mathcal{O}_{\mathbf{q}}(GL(N,\mathbb{C}))$ *with quantum control*

$$2_{\mathbf{q}} \oplus X \oplus \mathrm{Det}_{\mathbf{q}}(X)^{-1}$$

is an s^**-algebra.*

Similarly, s*-algebras $\mathcal{O}_{\mathbf{q}}(U(N)), \mathcal{O}_{\mathbf{q}}(T(N)), \mathcal{O}_{\mathbf{q}}(PD(N))$.

$\Rightarrow$ C*-algebras

$$C_{0,\mathbf{q}}(GL(N,\mathbb{C})), \quad C_{0,\mathbf{q}}(U(N)), \quad C_{0,\mathbf{q}}(T(N)), \quad C_{0,\mathbf{q}}(PD(N)).$$

Morphisms of s*-algebras

Definition

Let $\mathscr{A}, \mathscr{B}$ be s*-algebras with quantum controls X, Y.

s^*-morphism $\alpha : \mathscr{A} \to \mathscr{B}$ if $\exists F : \mathbb{R}^+ \to \mathbb{R}^+$ s.t.

$$\|\pi(Y)\| \leq M \quad \Rightarrow \quad \|\pi(\alpha(X))\| \leq F(M).$$

Proposition

An s^*-morphism $\alpha : \mathscr{A} \to \mathscr{B}$ completes to a non-degenerate

$$\alpha_0 : A_0 \to M(B_0)$$

$\mathscr{R}$-algebras and fields of C*-algebras

$\mathscr{R}$-algebras and fields of C*-algebras

Definition

Let $\mathscr{R}$ be a commutative s*-algebra.

We call $\mathscr{R}$-algebra any s*-algebra $\mathscr{A}$ with s*-morphism

$$\iota : \mathscr{R} \to \mathscr{Z}(\mathscr{A}) \subseteq \mathscr{A}.$$

Then R_0-algebra

$$\iota_0 : R_0 \to \mathscr{Z}(A_0),$$

and A_0 field of C*-algebras over $\Theta = \mathrm{Spec}(R_0)$.

Question

When is A_0 a continuous field of C-algebras?*

$\mathscr{R}$-algebras and fields of C*-algebras

A condition for continuity

Theorem (DC-Floré ('17))

Let $\mathscr{A}$ countably generated $\mathscr{R}$-algebra.

Let $(\mathscr{U}, \Delta, \Phi)$ type I-Hopf $\mathscr{R}$-algebra with coaction

$$\alpha : \mathscr{A} \to \mathscr{A} \underset{\mathscr{R}}{\otimes} \mathscr{U}.$$

Assume $\exists$ coinvariant central control and $\mathscr{B} := E(\mathscr{A}) \subseteq \mathscr{Z}(\mathscr{A})$,

$$E : \mathscr{A} \to \mathscr{A} \quad a \mapsto (\mathrm{id} \otimes \Phi)\alpha(a).$$

Assume $\mathrm{Spec}(B_0) \to \mathrm{Spec}(R_0)$ open, $E_{0,x} : A_{0,x} \to B_{0,x}$ faithful.

Then A_0 is a continuous field of C^*-algebras over $\mathrm{Spec}(R_0)$.

$\mathscr{R}$-algebras and fields of C*-algebras

Quantum decompositions revisited

Conjecture

The quantum Cholesky map induces an isomorphism

$$\chi^{\mathbf{q}}_{T,0} : C_{0,\mathbf{q}}(PD(N)) \cong C_{0,\mathbf{q}}(T(N)).$$

Corollary (DC-Floré ('17))

$C_{0,\mathbf{q}}(GL(N,\mathbb{C}))$, $C_{0,\mathbf{q}}(T(N))$ *and* $C_{0,\mathbf{q}}(PD(N))$ *are type* I, *and*

$$\chi^{\mathbf{q}}_{UT,0} : C_{0,\mathbf{q}}(GL(N,\mathbb{C})) \cong C_{0,\mathbf{q}}(U(N)) \underset{C_0((0,+\infty))}{\otimes} C_{0,\mathbf{q}}(T(N)).$$

Continuity of $C_{0,\mathbf{q}}(PD(N))$

Theorem (DC-Floré ('17))

The field of C-algebras $C_{0,\mathbf{q}}(PD(N))$ over $(0, +\infty)$ is continuous.*

Proof.

Apply previous theorem to

$$E = (\mathrm{id} \otimes \Phi) \, \mathrm{Ad}_{\mathbf{q}} : \mathcal{O}_{\mathbf{q}}(PD(N)) \to \mathcal{O}_{\mathbf{q}}(PD(N)),$$

using information on joint spectrum of C_k in CH-identity. □