A MODEL FLOW FOR SUBMANIFOLDS WITH CONSTANT CURVATURE

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One of the most basic pursuits in geometry is the understanding of shapes with least bending. In this talk, we interpret bending not as pure curvature but as a derivative of curvature (although linguistically it sounds odd, this is called the jerk), and take an energetic approach toward the analysis of shapes with least jerk. We propose a broad problem in the calculus of variations, on submanifolds with parallel mean curvature vector. As a first step, we study the problem in the geometrically mostly uninteresting case of curves in the plane. Here the gradient flow nevertheless challenges us to come up with new methods. First, we determine the set of equilibria – circles – through an elementary analysis of the Euler-Lagrange equation. Then, we define a scale-invariant energy and study the flow for small enough initial energy. After some effort, we prove convergence in this energetic neighbourhood of the flow to a round circle. Apart from energy estimates, the Lojasiewicz-Simon gradient inequality makes an appearance. We quite carefully establish the gradient inequality in our setting, which although simple, still requires some effort. This is joint work with Ben Andrews (ANU), James McCoy (UoN) and Valentina-Mira Wheeler (UOW).

Date: May 11, 2018.