HÖLDER REGULARITY OF ANISOTROPIC LEAST GRADIENT FUNCTIONS

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Consider the anisotropic least gradient problem in two dimensions

$$\min\{\int_\Omega \phi(Du), \quad u \in BV(\Omega), \quad u|_{\partial\Omega} = f\}$$

where $\Omega \subset \mathbb{R}^2$ is an open bounded set with Lipschitz boundary, $f \in C(\partial\Omega)$, and ϕ is an anisotropic norm on $\mathbb{R}^2$. As in the classical least gradient problem, existence and uniqueness of minimizers depend on the geometry on Ω. From here, there are two different scenarios:

(1) The unit ball $B_\phi(0,1)$ is strictly convex. In this case, we have existence and uniqueness of minimizers of the anisotropic least gradient problem for every boundary data $f \in C(\partial\Omega)$ if Ω is strictly convex. Furthermore, if Ω satisfies some form of uniform convexity, then we may obtain regularity estimates regardless of the regularity of ϕ; in particular, if $\partial\Omega \in C^2$ and the curvature is bounded from below, then $f \in C^{0,\alpha}(\partial\Omega)$ implies $u \in C^{0,\alpha/2}(\Omega)$ (the same as in the isotropic case).

(2) The unit ball $B_\phi(0,1)$ is not strictly convex. In this case existence of minimizers is obtained (for uniformly convex domains) using the regularity estimates from case (1). This gives us one minimizer with the same regularity estimates as in the previous paragraph. However, uniqueness of minimizers fails and the resulting minimizers may fail to be $W^{1,1}(\Omega)$ or $SBV(\Omega)$ even for smooth boundary data.