Total variation flow of curves

Michał Łasica
SBAI Department, Sapienza University of Rome
lasica@mimuw.edu.pl

Abstract

Let $(\mathcal{N}, g)$ be a complete, n-dimensional Riemannian submanifold in Euclidean space $\mathbb{R}^N$ and let I be a bounded interval. We introduce a natural notion of solution to the formal L^2-gradient flow of the total variation functional

$$TV_g(\boldsymbol{u}) = \int_I |\boldsymbol{u}_x|_g$$

for $\boldsymbol{u} \in H^1(0,T; L^2(I, \mathcal{N})) \cap L^\infty(0,T; BV(I, \mathcal{N}))$. Given any $\boldsymbol{u}_0 \in BV(I, \mathcal{N})$ whose jumps are not too large, we sketch the proof of existence of a solution for arbitrarily large $T > 0$.

An important ingredient of the proof is a *completely local* estimate

$$\int_A |\boldsymbol{u}_x(t, \cdot)|_g \leq \int_A |\boldsymbol{u}_{0,x}|_g$$

for any Borel $A \subset I$. This estimate seems to be new even in the case $\mathcal{N} = \mathbb{R}^n$.

The talk is based on a recent, unpublished joint work with L. Giacomelli.