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Weighted Hardy's inequalities and Kolmogorov-type operators

We give general conditions to state the weighted Hardy inequality

$$c\int_{\mathbb{R}^N}\frac{\varphi^2}{|x|^2}d\mu \leq \int_{\mathbb{R}^N}|\nabla\varphi|^2 d\mu + C\int_{\mathbb{R}^N}\varphi^2 d\mu, \quad \varphi\in C_c^\infty(\mathbb{R}^N),\ c\leq c_{0,\mu},$$

with respect to a probability measure $d\mu$. Moreover, the optimality of the constant $c_{0,\mu}$ is given. The inequality is related to the following Kolmogorov equation perturbed by a singular potential

$$Lu+Vu=\left(\Delta u+\frac{\nabla\mu}{\mu}\cdot\nabla u\right)+\frac{c}{|x|^2}u$$

for which the existence of positive solutions to the corresponding parabolic problem can be investigated. The hypotheses on $d\mu$ allow the drift term to be of type $\frac{\nabla\mu}{\mu}=-|x|^{m-2}x$ with $m>0$.

Joint work with A. Canale, F. Gregorio and C. Tacelli.