On decomposition of the identity operator into a linear combination of five projections

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Abstract

In [1], the authors studied families of orthogonal projections $P_1, \ldots, P_n$ in a separable complex Hilbert space H, for which

$$P_1 + \cdots + P_n = \lambda I, \quad \lambda > 0.$$

In particular, they described the set Σ_n of those $\lambda \in \mathbb{R}$, for which such a decomposition exists. While for $n \leq 4$, all such decompositions can be classified up to a unitary equivalence, for $n \geq 5$ such classification is possible only for λ in some discrete subset in Σ_n.

Next problem which arised here is to study families of projections, for which

$$\alpha_1 P_1 + \cdots + \alpha_n P_n = I, \quad 0 < \alpha_j < 1, \quad n = 1, \ldots, n,$$

in particular, to study the structure of the set Ω_n of admissible weights $(\alpha_1, \ldots, \alpha_n) \subset (0,1)^n$. Again, for $n \leq 4$, both the structure of Ω_n and the unitary classification of all such decomposions have been obtained, while for $n \geq 5$ only partial results are known. Notice that such decompositions arise in frame theory as tight fusion frames.

We discuss some recent results on the structure of sets Ω_n, $n \geq 5$, especially focusing our attention at the case $n = 5$. In particular, we show that if $\alpha_5 < \epsilon$ for small ϵ, then there exists $\Omega_4 \ni \beta = (\beta_1, \beta_2, \beta_3, \beta_4)$ with $\max_{1 \leq j \leq 4} |\alpha_j - \beta_j| < \epsilon$.

This talk is based on joint results with Slavik Rabanovich.

[1] Kruglyak, S. A., Rabanovich, V. I., Samolenko, Yu. S. On sums of projections. *Funct. Anal. Appl.* **36** (2002), no. 3, 182–195.