ON THE NONLOCAL DIRICHLET PROBLEM

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We prove a classical representation formula for distributional solutions of the nonlocal Dirichlet problem generated by translation-invariant integrodifferential operators of the form

$$\mathcal{L}u(x) = \lim_{\epsilon \to 0} \int_{|y|>\epsilon} \big(u(x+y) - u(x)\big)\nu(y)\,\mathrm{d}y,$$

where $\nu\colon \mathbb{R}^d \setminus \{0\} \mapsto [0,\infty)$ is a radial, non-increasing function satisfying the condition $\int_{\mathbb{R}^d} \big(1 \wedge |x|^2\big)\nu(x)\,\mathrm{d}x < \infty$, and study the question under which assumptions distributional solutions are twice differentiable in the classical sense. Sufficient conditions and counterexamples are provided. In particular, we show that the distributional solution of the equation

$$\begin{cases} \Delta^{\alpha/2} u &= f \quad \text{in } B_1, \\ u &= 0 \quad \text{in } B_1^c, \end{cases}$$

where $\alpha \in (0,2)$ and $f \in C^{2-\alpha}(B_1)$, does not have to be in $C^2(B_1)$.

References

[1] T. Grzywny, M. Kassmann, and Ł. Leżaj. Remarks on the nonlocal Dirichlet problem. Preprint, 2018, arXiv:1807.03676.