ON THE AFFINE RECURSION IN DIMENSION ≥ 2 IN THE CRITICAL CASE

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We fix $d \geq 2$ and denote $\mathcal{S}$ the semi-group of $d \times d$ matrices with non negative entries. We consider a sequence $(A_n, B_n)_{n \geq 1}$ of i. i. d. random variables with values in $S \times (\mathbb{R}^+)^d$ and study the asymptotic behavior of the Markov chain $(X_n)_{n \geq 0}$ on $(\mathbb{R}^+)^d$ defined by:

$$\forall n \geq 1, \qquad X_{n+1} = A_{n+1} X_n + B_{n+1},$$

where X_0 is a fixed random variable.

We assume that the Lyapunov exponent of the matrices A_n equals 0 and prove, under quite general hypotheses, that there exists a unique (infinite) Radon measure λ on $(\mathbb{R}^+)^d$ which is invariant for the chain $(X_n)_{n \geq 0}$.

The existence of λ relies on a recent work by T.D.C. Pham about fluctuations of the norm of product of random matrices.

Its unicity is a consequence of a general property, called "local contractivity", highlighted about 20 years ago by M. Babillot, Ph. Bougerol et L. Elie [**?**] then S. Brofferio [**?**] in the case of the affine recursion in dimension 1. This property has been extensively studied for general iterated function systems by M. Peigné et W. Woess [**?**]; as far as we know, the affine recursion we present here is the first example in dimension ≥ 2 where this weak contractivity phenomenon is described.

References

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[4] Peigné M. & Woess W. *Stochastic dynamical systems with weak contractivity I. Strong and local contractivity* , Colloquium Mathematicum 125, 1–54, 2011.