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The covariant functoriality of graph algebras

In the standard category of directed graphs, morphisms respect the lengths of paths. However, this requirement is way too strong to present natural *-homomorphisms between graph C*-algebras as covariantly induced from morphisms of graphs, so we define a new category of directed graphs where morphisms are defined as appropriate maps between spaces of finite paths, and called path morphisms of graphs. For instance, the natural *-homomorphism from the minimal unitization of compact operators to the Toeplitz algebra (shrinking the boundary of the Klimek-Lesniewski quantum disc to obtain the standard Podleś quantum sphere) is covariantly induced by a morphism mapping edges to ever longer paths. The goal of this talk is to unravel how to restrict path morphisms of graphs, so that they covariantly induce homomorphisms of path algebras, Cohn path algebras, and Leavitt path algebras, respectively. Thus we obtain a new category of directed graphs with a covariant functor to the category of graph C*-algebras, and substantially extend the class of path morphisms of graphs shown to admit such a covariant functor by Chirvasitu, Hajac and Tobolski.