Matthias Schötz

Sums of squares in Leavitt path *-algebras and beyond

I will discuss how the theory of *-algebras of operators is a form of non-commutative real algebraic geometry. The key notion here is the concept of an ordered *-algebra, i.e. a *-algebra equipped with a quadratic module of "positive" elements. C*-algebras are particularly well-behaved examples of ordered *-algebras, but there are more examples: In contrast to C*-algebras, an ordered *-algebra is not necessarily (uniformly) bounded and its order does not necessarily arise from the operator order in *-representations. This in particular allows to easily construct ordered *-algebras from generators and relations, which is a natural procedure for many classes of examples, including the Leavitt path *-algebras constructed out of directed graphs. The question arises how such algebraic constructions relate to their analytic counterparts, like graph C*-algebras: We need to study algebraic certificates of (uniform) boundedness and of positivity, i.e. (non-commutative) Positivstellensätze. I will give an overview about some selected results in this direction, for Leavitt path *-algebras and beyond.