LOOPS'23: NONASSOCIATIVE ALGEBRA
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PREFACE

This volume comprises various papers presented during the international conference LOOPS'23 which took place in Będlewo at the Mathematical Research and Conference Center of the Institute of Mathematics of the Polish Academy of Sciences, between 25 June and 2 July 2023. The LOOPS conferences have been held every 4 years since 1999. They cover all aspects of nonassociative mathematics, including quasigroups, loops, Latin squares, Lie algebras, Jordan algebras, octonions, quandles, biracks, and applications. The conferences intend not only to highlight new relevant results in algebra and geometry, but also to foster connections to combinatorics, group theory, knot theory, cryptography, and physics. Previous LOOPS conferences were held in Prague, Czech Republic; Trest, Czech Republic; Ohrid, North Macedonia; and Budapest, Hungary.

Nonassociative algebraic structures have a well-established tradition in mathematics, due both to their intrinsic interest and to their applications in other areas. A well-known example is given by the octonions, which have applications in fields such as string theory, special relativity, and quantum logic.

The oldest and most developed discipline of nonassociative algebra is the theory of quasigroups, including loops which are quasigroups with an identity element. In the finite case, a quasigroup is a magma such that the multiplication table of the binary operation is a Latin square, where in each row and each column, each element appears exactly once. Starting from early work going back to Euler, the theory was developed in the 1930's in works of Sushkevich, Moufang, Bol, Murdoch, and others. The topic is covered by several books that study various aspects of the theory.

Idempotent left distributive left quasigroups have become popular under the name of quandles – algebraic structures whose axioms encode the three Reidemeister moves of knot theory. Quandles thus bring together aspects of topology, abstract algebra and combinatorics, and the main motivation behind the theory is finding knot invariants that are easy to compute. In fact, Joyce and Matveev proved in 1982 that the knot quandle is a complete invariant up to orientation: two diagrams give rise to isomorphic quandles if and only if the diagrams represent the same knot regardless of orientation.

In a broader perspective, quandles and racks (non-idempotent quandles) provide set-theoretical solutions to the quantum Yang–Baxter equation – a fundamental equation occurring in integrable models in statistical mechanics and quantum field theory. It is known that there is a one-to-one correspondence between (involutive) solutions of the Yang–Baxter equation and (involutive) biracks – algebras combining two one-sided quasigroups satisfying some additional identities.

Along with their applications to other branches of mathematics, quasigroups also have direct applications in practice. For example, Latin squares have played an important role in scheduling and the statistical design of experiments. Most applications of quasigroups in cryptography relate to their various kinds of inverse property.

Finally, there is fruitful interaction between nonassociative mathematics and automated deduction. Several important results in nonassociative algebra have been initially obtained by automated deduction, and conversely, problems from nonassociative algebra have led to advances in the development of automated theorem provers.

The LOOPS'23 conference consisted of two parts: the workshop – with a series of tutorial lectures aimed at both active researchers and graduate students; and the conference proper – with four major plenary lectures and twenty-seven short, contributed talks. A highlight of the conference was the presentation by Dimitri Kanevsky (Research Scientist at Google, Mountain View, California, USA) of his positive solution to Manin's problem on the existence of nonassociative Moufang loops of classes of points on cubic hypersurfaces, which had remained open for over 50 years.

The volume contains articles reflecting current developments in various branches of nonassociative algebra: Moufang loops, general loops, loops related with the Yang–Baxter equation, solutions of the Yang–Baxter equation, knot theory and Lie theory.

The conference was organized by the Stefan Banach International Mathematical Center; the Institute of Mathematics of the Polish Academy of Sciences; and the Faculty of Mathematics and Information Science, Warsaw University of Technology.

The Scientific Committee consisted of:

- Janusz Grabowski, Institute of Mathematics, Polish Academy of Sciences, Poland;
- Alexander Grishkov, Universidade de São Paulo, Brazil;
- Tomáš Kepka, Charles University, Czech Republic;
- Victoria Lebed, Université de Caen, France;
- Gábor Nagy, Budapest University of Technology, Hungary;
- Alex Nowak, Howard University, USA;
- Simona Samardjiska, Radboud University, The Netherlands;
- David Stanovský, Charles University, Czech Republic;
- Jonathan Smith, Iowa State University, USA;
- Izabella Stuhl, Pennsylvania State University, USA;
- Petr Vojtěchovský, University of Denver, USA.

The Organizing Committee consisted of:

- Katarzyna Grabowska, University of Warsaw;
- Janusz Grabowski, Institute of Mathematics, Polish Academy of Sciences;
- Přemysl Jedlička, Czech University of Life Sciences;
- Agata Pilitowska, Warsaw University of Technology;
- Anna Zamojska-Dzienio, Warsaw University of Technology.

More information about the conference is posted on the conference web-site: https://www.impan.pl/en/activities/banach-center/conferences/23-loops.