

PREFACE

The *Gdańsk-Kraków-Łódź-Warszawa workshop in Singularity Theory – a special session dedicated to the memory of Stanisław Łojasiewicz* was held in December 2022 at the Banach Center, Institute of Mathematics of the Polish Academy of Sciences, Warsaw. Its goal was to commemorate the 20th anniversary of the death of Stanisław Łojasiewicz by bringing together world-class experts in analytic geometry and singularity theory. The meeting took place in a hybrid form, both on-site and online, and gathered researchers from sixteen different countries. It was structured around two mini-courses:

- Symplectic monodromy “at radius 0” and equimultiplicity of families of hypersurfaces with constant Milnor number (Javier Fernández de Bobadilla)
- Gradient inequalities, generalizations and quantitative aspects (Krzysztof Kurdyka)

and ten invited lectures:

- Partial desingularization (Edward Bierstone)
- Lipschitz geometry of real surfaces – general overview (Lev Birbrair)
- Poincaré–Hopf theorem à la Lipschitz – inspired by Marie-Hélène Schwartz and Tadeusz Mostowski (Jean-Paul Brasselet)
- Regularity in analysis and geometry and the influence of Łojasiewicz (Tobias Colding)
- Lipschitz geometry of abnormal definable surface germs (Andrei Gabrielov)
- Łojasiewicz inequalities in a certain class of smooth functions (Hà Huy Vui)
- Approximation and homotopy in real algebraic geometry (Wojciech Kucharz)
- Motivic, logarithmic, and topological Milnor fibrations (Adam Parusiński)
- On the main scientific achievements of Stanisław Łojasiewicz (Wiesław Pawłucki)
- The extra-nice dimensions (Maria Aparecida Soares Ruas)

There were also 11 contributed talks.

The present volume offers a selection of articles covering various topics discussed during the workshop, including Lipschitz geometry, Łojasiewicz inequalities, o-minimal structures and singularities of curves and surfaces. It aims to reflect current aspects of the research in these areas as well as to highlight Łojasiewicz’s influence on them.

The volume starts with a beautiful overview “Łojasiewicz gradient inequality after half a century” by Bronisław Jakubczyk on the resonances of the Łojasiewicz gradient inequality in the study of differential equations, geometric analysis and probability theory. It can be seen as an invitation to open new perspectives in research for specialists in singularities and geometry as well as in PDE’s.

Four articles are based on lectures given during the workshop: “Poincaré–Hopf theorem for singular analytic varieties – The Marie-Hélène Schwartz ideas in the Lipschitz framework”, where Jean-Paul Brasselet, Tadeusz Mostowski and Thủy Nguyễn Thị Bích explain how the classical Marie-Hélène Schwartz construction of a stratified radial vector field in the context of Whitney stratifications can be simplified in the framework of Lipschitz stratifications, and prove a Poincaré–Hopf theorem for Lipschitz radial vector fields in this framework; “Łojasiewicz inequalities in a certain class of smooth functions”, in which Hà Minh Lam and Hà Huy Vui investigate Łojasiewicz inequalities for non-degenerate smooth functions satisfying the so-called Kamimoto–Nose condition; “KL-inequality for definable maps”, where Krzysztof Kurdyka proves a version of the Kurdyka–Łojasiewicz inequality for a mapping definable in an o-minimal structure with values in \mathbb{R}^k , $k > 1$; and “Non-Archimedean resonances of Łojasiewicz inequalities”, in which Krzysztof Nowak presents a survey on Łojasiewicz inequalities and their applications in the realm of non-Archimedean geometry.

There are two articles concerning the study of singularities of curves using the techniques of the Newton diagram: “A formula for the degree of singularity of plane algebroid curves” by Evelia R. García Barroso and Arkadiusz Płoski, and “On the Milnor number of a polar curve of a generic plane branch with one characteristic exponent” by Mateusz Masternak and Michał Zakrzewski. In both articles, some formulas for the Milnor number are given. Still in the classical theory of singularities, we present a paper “A study of the irreducibility of a generic hyperplane section through a normal singularity” by Maria Rosario Gonzalez-Dorrego, where a generalization of a theorem of Artin from rational to normal singularities is formulated.

In the article “Tame topology and non-integrability of dynamical systems”, Zbigniew Hajto and Rouzbeh Mohseni investigate the concept of (non-)integrability of dynamical systems within the framework of differential Galois theory.

Wojciech Kucharz and Krzysztof Kurdyka in the paper “On a theorem of Forelli in a non-standard setting” give a result analogous to the classical Forelli theorem for functions defined over the algebraic closure $C = R(\sqrt{-1})$ of a given uncountable real closed field R .

Finally, in “Intersection number of a map with the set of matrices of positive corank”, Iwona Krzyżanowska and Aleksandra Nowel give an effective method to compute the intersection number mod 2 of a polynomial map with the set of matrices of positive corank.

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