Homogenization of a certain class of Itô-Lévy processes with stationary and ergodic coefficients

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In our talk we consider the problem of homogenization of the solutions of Itô-Lévy type stochastic differential equations

$$\begin{aligned} X^x(t;\omega) \,=\, & x + \int_0^t \int_{\mathbb{R}^d} \gamma\left(X^x(s-;\omega), z;\omega\right) \tilde{N}(ds,dz) \\ & + \int_0^t \sigma\left(X^x(s-;\omega);\omega\right) dB(s), \end{aligned} \tag{1}$$

where $\gamma : \mathbb{R}^d \times \mathbb{R}^d \times \Omega \to \mathbb{R}^d$, $\sigma : \mathbb{R}^d \times \Omega \to \mathbb{R}^{d\times d}$ are sufficiently regular random fields defined over a probability space $(\Omega, \mathcal{F}, \mu)$. Here $\tilde{N}(dt,dz) = N(dt,dz) - \nu(dz)dt$, where $B(t)$ and $N(dt,dz)$ are a d-dimensional, standard Brownian motion and a random Poisson measure with the spatial intensity $\nu(dz)$, that are independent of each other and also independent of the fields $\gamma(x,z;\omega)$ and $\sigma(x;\omega)$.

We prove the existence of the limits of the laws of scaled processes $\varepsilon X^x(t\varepsilon^{-\alpha};\omega)$, as $\varepsilon \to 0+$, with an appropriate choice of the exponent $\alpha > 0$. We shall distinguish two cases: first, when the random variable $\int_{\mathbb{R}^d} |\gamma(0,z;\omega)|^2 \,\nu(dz)$ is L^p integrable, with a sufficiently large p. Then, under the diffusive scaling ($\alpha = 2$) the processes $\varepsilon X^x(t\varepsilon^{-2};\omega)$ converge in law, as $\varepsilon \to 0+$, to a Brownian motion.

Our second result concerns the case when the sizes of the jumps are so large that $\int_{\mathbb{R}^d} |\gamma(0, z; \omega)|^2 \, \nu(dz) = +\infty$ a.s. Then, under some assumptions about the field $\gamma(0, z; \omega)$ and a suitable choice of the exponent $\alpha \in (0, 2)$, the scaled processes $\varepsilon X^x(t\varepsilon^{-\alpha}; \omega)$ converge to some Levy process.

To prove the results we formulate an extension of the Alexandrov-Bakelman-Pucci estimates in the case of non-local operators. This result is of independent interest.