HARDY SPACES FOR FOURIER INTEGRAL OPERATORS

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It is well known that the wave operators $\cos(t\sqrt{-\Delta})$ and $\sin(t\sqrt{-\Delta})$ are not bounded on $L^p(\mathbb{R}^n)$, for $n \geq 2$ and $1 \leq p \leq \infty$, unless $p = 2$ or $t = 0$. In fact, for $1 < p < \infty$ these operators are bounded from $W^{(n-1)|\frac{1}{p}-\frac{1}{2}|,p}(\mathbb{R}^n)$ to $L^p(\mathbb{R}^n)$, and this exponent cannot be improved. This phenomenon is symptomatic of the behavior of Fourier integral operators, a class of operators which includes the solution operators to smooth variable-coefficient wave equations.

In this talk, I will introduce a class of Hardy spaces $\mathcal{H}^p_{FIO}(\mathbb{R}^n)$, for $1 \leq p \leq \infty$, on which suitable Fourier integral operators are bounded. These spaces also satisfy Sobolev embeddings that allow one to recover the optimal boundedness results for Fourier integral operators on $L^p(\mathbb{R}^n)$.

In fact, the invariance of these spaces under Fourier integral operators allows for iterative constructions that are not possible when working directly on $L^p(\mathbb{R}^n)$. I will also mention the connection of these spaces to the local smoothing conjecture.

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