Title: Łojasiewicz's gradient inequality solving problems in PDEs, geometric analysis, optimization, and ... artificial intelligence

Abstract: In 1963 Stanisław Łojasiewicz proved that for an arbitrary real analytic function f defined in a neighbourhood of $0 \in \mathbb{R}^n$ such that $f(0) = 0$ and $f(x) \geq 0$ there exists $\theta \in (0, 1/2]$ such that

$$|f(x)|^\theta \leq C\|\nabla f(x)\|$$

for x close to 0, with θ depending on the singularity of f at 0 and $C > 0$. He used the inequality for proving that any trajectory of the gradient flow $\dot{x} = -\nabla f(x)$ starting in a neighbourhood of 0 converges to a point x_∞ such that $f(x_\infty) = 0$ and $\nabla f(x_\infty) = 0$.

We will briefly overview how in the next 60 years the inequality, and its variant proved by L. Simon, turned out powerful tools in studying asymptotic behaviour of solutions to PDEs, problems in geometric analysis involving singularies of solutions and asymptotic cones of Riemannian metrics, studying Ricci flow, heat Yang-Mills flow, and others. Another branch of applications includes convergence analysis of gradient-like algorithms used in various optimization problems met in applications including stochastic gradient descent in machine learning and (gradient descent) deep learning by back-propagating errors of Geoffrey E. Hinton et. al. (Nobel Prize in Physics 2024).