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Coexistence of vertically and horizontally transmitted parasite strains in a simple SI type model

We study an SI type endemic model with one host and two parasite strains with complete cross protection between the strains. We assume that one strain is exclusively vertically transmitted and the other strain is horizontally (and possibly also vertically) transmitted. We assume that each strain reduces fertility and/or increases mortality of infected hosts. Our model consists of just three ordinary differential equations. We use the mathematical theory of persistence to show that the (exclusively) vertically transmitted strain that would go extinct by itself can persist by protecting the host against the more virulent horizontally transmitted strain [2]. There are two more interesting properties of our model. First, the ratio of horizontal to vertical transmission decreases if the coefficient of horizontal transmission increases, contrary to what one might expect [1]. Second, the equilibrium where both parasite strains coexist is always locally asymptotically stable if the horizontal transmission is of density-dependent (mass-action) type, but can lose its stability and give rise to a limit cycle if the horizontal transmission is of frequency-dependent (standard) type [3].

REFERENCES

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