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The development of fingers in solid tumors

We consider a solid tumor in a region which is modeled either as a porous medium (by Darcy's law) or as fluid-like tissue (by Stokes equation). We assume that the proliferating and dying cells move around with velocity v in a way that keeps their density constant in the tumor region $D(t)$. The nutrient concentration and the velocity v satisfy a system of PDEs in $D(t)$. The aggressivity of the tumor is represented by a parameter μ which relates nutrient concentration to proliferating rate of cells. It is shown that there is a stationary spherically symmetric solution of radius R which depends on some of the model parameters but not of μ. We prove that this solution is asymptotically stable for $\mu > \mu_*$ and there exist infinite number of branches of stationary solutions with arbitrarily large number of fingers, indicating the onset of metastasis. We also prove that the fluid-like tumor develops more fingers than the tumor with porous medium consistency.