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Network reconstruction from nonstationary spike trains

Existing approaches to the problem of extracting neuronal connectivity from spike data [1,2] assume that the network is in a stationary state, which it is not in many experiments. Here we describe a method for inferring both the network connectivity and the time-dependent external drive that causes the nonstationarity.

Consider an experiment in which the neurons recorded are subjected repeatedly to a potentially unknown external input (such as would arise from sensory stimulation). The spikes are assumed to be binned in time and represented by a binary array: $S_i(t,r) = 1$ indicates a spike and $S_i(t,r) = -1$ indicates no spike by neuron i in time bin t of repetition r of the measurement. We fit these data to the simplest kind of binary stochastic model: At time step t of repetition r, each formal neuron receives a net input, $H_i(t,r) = h_i(t) + \sum_j J_{ij} S_j(t,r)$, and it takes the value $+1$ at the next step with a probability given by a logistic sigmoidal function $1/[1 + \exp(-H_i(t,r))]$ of $H_i(t,r)$. Maximizing the likelihood of the data leads to learning rules

$$\text{(1)} \qquad \delta h_i(t) = \eta_h \left\{ \langle S_i(t+1,r) \rangle_r - \langle \tanh[H_i(t,r))] \rangle_r] \right\}$$

$$\text{(2)} \qquad \delta J_{ij} = \eta_J \left\{ \langle S_i(t+1,r) S_j(t,r) \rangle_{rt} - \langle \tanh[H_i(t,r)] S_j(t,r) \rangle_{rt} \right\}$$

for the model parameters – the couplings J_{ij} and external inputs $h_i(t)$. For weak coupling or densely connected networks, faster alternative algorithms are possible [3], based on expanding (1) and (2) around mean-field and TAP [4] equations for $m_i(t) = \langle S_i(r,t) \rangle_r$.

Here we present results of applying both this and methods assuming stationarity to (1) data generated by the stochastic model itself (the realizable case), (2) data from a realistic computational model of a small cortical network, and (3) data recorded from salamander retina under visual stimulation. We show that, in all three cases, performing the reconstruction assuming stationarity systematically overestimates the couplings in the network: the algorithms effectively invent fictitious couplings to explain stimulus-induced correlations. The nonstationary treatment outlined above enables us to find, for sufficient data, the correct (weaker) couplings and to extract the time-dependence of the external input.

References

[1] E Schneidman, M Berry, R Segev and W Bialek, *Weak pairwise correlations imply strongly correlated networks states in a neural population*, Nature **440** 1007-1012 (2006).

[2] Y Roudi, J Tyrcha and J Hertz, *Ising model for neural data: Model quality and approximate methods for exctracting functional connectivity*, Phys Rev E **79** 051915 (2009).

[3] Y Roudi and J Hertz, *Mean Field Theory for Nonequilibrium Network Reconstruction*, Phys Rev Lett **106** 048702 (2011).
[4] D Thouless et al, *Solution of 'solvable model of a spin glass'*, Phil Mag **35** 593-601 (1977).