Zofia Jones
Mathematical Sciences, University of Nottingham, Nottingham, NG7 2RD, UK
e-mail: pmxzj1@nottingham.ac.uk

Helfrich Energy Model of the Phagocytosis of a Fibre

CNTs are a form of High Aspect Ratio Nanoparticles (HARN). Their radius is typically of only a few nanometres (10^{-9}) while their length can be on the micron scale (10^{-6}). Their shape has been found to make their removal from the lung surface on inhalation by macrophages especially difficult. This is widely regarded as a key mechanism of toxicity [1] [2]. Frustrated phagocytosis leads to scarring and granuloma formation which impairs the function of the lung.

Following the precendent set by Helfrich and Deuling [3] [4], the free energy of a cell membrane is taken to be given by

$$\mathrm{F} = \underbrace{\int_V \Delta_p}_{\text{Volume Energy}} + \underbrace{\int_S \lambda}_{\text{Surface Energy}} + \underbrace{\int_S (\text{mean curvature} - c_0)^2}_{\text{Helfrich Energy}}$$

The Helfrich energy was introduced in [3] to quantify the energy associated with a cell membrane of a particular shape. It is often referred to as the bending energy. The spontaneous curvature c_0 takes into account the natural curvature of a cell membrane due to proteins in the lipid bilayer and the cytoskeleton.

For a given set of boundary conditions, the shape of a the cell membrane is found by solving the associated Euler-Lagrange equations. The topology of the surface is restricted to that of a surface of rotation around an axis which is taken to be the axis of a fibre. Due to singularities in these Euler-Lagrange equations, the problem is a boundary value problem rather than an initial value problem.

The solutions of this energy minimisation problem in [4] correspond to solutions in the limit of a vanishing radius of the cell on a fibre problem. Boundary conditions specific to the cell on a fibre problem are introduced. These boundary conditions can be chosen to ensure that the boundary terms of the first variation in the free energy are set to zero. They can also be chosen to fix the contact angle of the cell membrane with the fibre surface.

It is assumed that the shape of a lipid membrane which has successfully engulfed a particle will be energetically stable, in order to conserve the limited resources of a macrophage. This does not take into account the energy required to remodel the cytoskeleton for the cell to reach this shape. However, the bending energy associated with cell membranes of increasing length can be used to suggest the amount of energy required in this dynamical process.

References

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