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Dynamics of plant water transport derived from applying an optimisation scheme to Soil-Plant-Atmosphere-Continuum

In Central Europe, plant transpiration injects more than 40% of precipitation back into the atmosphere. Thus, plants play an important role in the exchange of water between soil and atmosphere.

Plants can actively open and close their leaf openings ("stomata"), gateways for incoming carbon dioxide molecules to be processed by photosynthesis as well as for outgoing water vapour. Since both gas species use the same pathways, the majority of terrestrial plants has to compromise between the conflicting tasks of (i) minimising transpiration (in order to avoid water stress and wilting) and (ii) maximising assimilation of carbohydrates (which constitute both building material and energy source of plants).

Plants deal with this conflict by regulating leaf gas exchange (via stomatal aperture) according to soil moisture and the diurnal cycles of temperature, insolation and relative humidity. The (physiological) details of this regulation mechanism are largely unknown. Nonetheless, it is possible, to emulate the actual plant gas exchange by a mathematical optimisation scheme ([1], [2], [3]): Optimum stomatal conductance as a function of time is determined by requiring that the assimilates assembled during one day accumulate to a maximum, being subject to the constraint that the quantity of water transpired during this time span equals a given amount. The diurnal variations of temperature, insolation and relative humidity have to be prescribed.

The calculus of variation subject to constraints introduces a Lagrangian multiplier whose value cannot be determined in the usual way, due to an intractable integral. Application of the continuity equation to the water current through soil, plant roots and xylem allows, however, to express the Lagrangian multiplier in terms of soil properties, tree anatomy and tree physiologic restrictions.

Applications of this model encompass the reconstruction of palaeo-environment from fossilised plant leaves and the predictions of the impact of changing atmospheric CO_2-level on climate ([4]). Redistribution of precipitation between soil (run-off and ground water) and atmosphere (transpiration) due to modified stomatal action caused by changing atmospheric CO_2-content can also be assessed.

References

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