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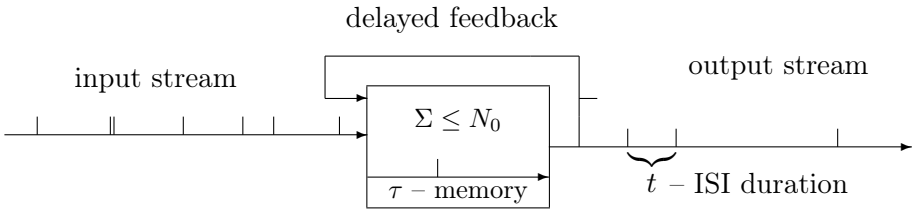
Delayed feedback results in non-markovian statistics of neuronal firing

The output inter-spike intervals (ISI) statistics of a single neuron with delayed feedback is considered. The construction is driven externally with Poisson stream of excitatory impulses. Via the feedback line, neuron's output impulses are fed back to its input with a fixed time delay. We consider cases of both excitatory and inhibitory neuron. Namely, in the first case, the neuron receives excitatory impulses both from the driving Poisson stream and from its own output stream through the feedback line. In the second case, apart from the external Poisson excitation, the delayed self-inhibition is present. For analytical derivation, we take binding neuron (BN) model [1].



We obtain exact analytical expressions for the single-ISI conditional probability density $P(t_2 \mid t_1)$, which gives the probability to obtain an output ISI of duration t_2 provided the previous ISI duration was t_1, and for the double-ISI conditional probability density $P(t_2 \mid t_1, t_0)$.

It turns out, that $P(t_2 \mid t_1)$ does not reduce to the output ISI probability density $P(t_2)$, found before. This means, that firing statistics is non-renewal one even in the simplest possible neuronal network. Moreover, we prove exactly, that $P(t_2 \mid t_1, t_0)$ cannot be reduced to $P(t_2 \mid t_1)$, the dependence on t_0 cannot be eliminated. This exactly means that ISIs stream does not possess Markov property.

Also, we introduce the conditional probability density $P(t_{n+1} \mid t_n, \ldots, t_1, t_0)$. It is proven exactly, that $P(t_{n+1} \mid t_n, \ldots, t_1, t_0)$ does not reduce to $P(t_{n+1} \mid t_n, \ldots, t_1)$ for any $n \geq 0$. This means that the output ISIs stream cannot be represented as Markov chain of any finite order.

We conclude, that the delayed feedback presence causes non-markovian behavior of neuronal firing statistics for both excitatory and inhibitory neurons. We suggest, that interpretation of experimental records of spiking activity should take this fact into account.

References

[1] A.K. Vidybida, *Neuron as time coherence discriminator.* Biol. Cybern. **74** 539–544 (1996).
[2] K.G. Kravchuk, A.K. Vidybida, *Delayed feedback causes non-Markovian behavior of neuronal firing statistics.* arXiv:1012.6019v2.