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Resistance Distance and Relatedness on an Evolutionary Graph

When investigating evolution in structured populations, it is often convenient to consider the population as an evolutionary graph – individuals as nodes, and their relations as edges. There has, in recent years, been a surge of interest in evolutionary graphs, especially in the study of the evolution of social behaviors ([5],[6]). An inclusive fitness framework is best suited for this type of study [2]. An expression for the genetic similarity between individuals residing on the graph is required for inclusive fitness calculations. This has been a major hindrance for work in this area as highly technical mathematics are often required [1]. In this presentation, I will derive a recent result [4] that links genetic relatedness between haploid individuals on an evolutionary graph to the resistance between vertices on a corresponding electrical network. Specifically, if R_{ij} be the relatedness and γ_{ij} the resistance distance [3] both between individuals i and j on a transitive graph G with N vertices each of degree k. Then,

$$R_{ij} = 1 - \frac{\gamma_{ij}}{\gamma_{ave}}.$$

An example that demonstrates the potential advantage of this result over contemporary approaches will be provided. I will discuss some new insights into the relatedness concept brought about by this result and mention possible directions for future investigation.

References

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