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The evolving surface finite element method (ESFEM) for pattern formation on evolving biological surfaces

In this talk we propose models and a numerical method for pattern formation on evolving curved surfaces. We formulate reaction-diffusion equations [4] on evolving surfaces using the material transport formula, surface gradients and diffusive conservation laws [1]. The evolution of the surface is defined by a material surface velocity. The numerical method is based on the evolving surface finite element method (ESFEM) [2, 3]. The key idea is based on the approximation of Γ by a triangulated surface Γ_h consisting of a union of triangles with vertices on Γ. A finite element space of functions is then defined by taking the continuous functions on Γ_h which are linear affine on each simplex of the polygonal surface. To demonstrate the capability, flexibility, versatility and generality of our methodology we present results for uniform isotropic growth as well as anisotropic growth of the evolution surfaces and growth coupled to the solution of the reaction-diffusion system. The surface finite element method provides a robust numerical method for solving partial differential systems on continuously evolving domains and surfaces with numerous applications in developmental biology, tumour growth and cell movement and deformation.

References

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