Alistair Middleton
Center for Biological Systems Analysis, Albert-Ludwigs-Universität Freiburg i.Br., Habsburgerstrasse 49 79104 Freiburg i.Br.
e-mail: alistair.middleton@zbsa.de
Christian Fleck
Center for Biological Systems Analysis, Albert-Ludwigs-Universität Freiburg i.Br., Habsburgerstrasse 49 79104 Freiburg i.Br.
e-mail: Christian.Fleck@fdm.uni-freiburg.de
Ramon Grima
The University of Edinburgh, School of Biological Sciences, C.H. Waddington Building, Room 3.03, King's Buildings Campus, Mayfield Road, Edinburgh, Scotland
e-mail: Ramon.Grima@ed.ac.uk

From particles to PDEs: continuum approximations to models of cellular migration

Cell migration is a fundamental process in biology. Examples range from the development of multi-cellular organisms, through to the emergence of complex spatial patterns in bacterial populations. Mathematical models of cell migration can help increase our understanding of the underlying biology. However, the models that capture the molecular scale interactions are typically rather complex and can be difficult to analyze. Here, we explore this problem by developing a model based on Langevin dynamics, whereby short-range intercellular interactions are represented using an appropriate potential function. Following Newman and Grima (2004), we obtain a mean field approximation to our model, this being an integro-partial differential equation. By exploiting the biologically plausible limit of intercellular interactions occurring on infinitesimally small length scales, we derive a system of partial differential equations that can approximate the mean-field behaviour of the original Langevin model and and is amenable to analysis. We will show how the molecular scale details (represented by our choice of interaction potential) are reflected in the PDE approximation. An analysis of the resulting patterns will be given. Relevant applications, such as cell-sorting and chemotaxis, will also be discussed.