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On the Interspike Times of two Coupled Neurons

Stochastic Leaky Integrate and Fire models describe the evolution of the membrane potential $\{X_t\}_{t\geq 0}$ through the Stein equation

$$\begin{cases} dX_t = -\frac{X_t}{\tau}dt + adN_t^+ + idN_t^- \\ X_0 = x_0 \end{cases}.$$

Here, $a > 0, i < 0$ are constants representing excitatory and inhibitory inputs, τ is the membrane time constant and x_0 is the resting potential. Furthermore, $\left\{N_t^+\right\}$ and $\left\{N_t^-\right\}$ are two independent Poisson processes of rates $\lambda > 0$ and $\beta > 0$, respectively. The release of a spike corresponds to the first time when the membrane potential attains a threshold value $S > x_0$. After a spike, the membrane potential is reset to its resting value and the process restarts its evolution until a time t_{max}. The Interspike Intervals (ISI) are modeled through the random variables $T = \inf\{t : X_t > S\}$. In the seventies, the difficulty of the first passage time problem for the Stein process has motivated the introduction of diffusion limits for its equation. As result, an Ornstein-Uhlenbeck process is obtained. It models the sub-threshold membrane potential dynamics and it has developed the study of the input-output relationships of a single neuron.

However, one should consider two or more dependent neurons to study the transmission on information in a network. Here, we extend the Stein process to the case of k neurons, modeling its spiking activity. For this aim, we prove the convergence of a k-dimensional Stein process to a k-dimensional Ornstein-Uhlenbeck one. We also prove the weak convergence of their ISIs.

In the two dimensional case, we numerically determine the joint distribution of the ISIs of the two neurons. Finally, we illustrate some results on the dependencies of these times.