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A New Mathematical Model for combining Transport and Degradation in the Small Intestine

The small intestine is responsible for the major part of feedstuffs digestion in the gastrointestinal tract. Several models have been developed for representing the digestion of a bolus in the small intestine ([1], [2], [3]). This work tries to go further in modeling these phenomena by representing a simultaneous model for degradation and absorption of feedstuffs and their transport in the intestinal lumen. Specifically, we determine the position of the bolus and the proportion of the constituents at each time. In the first part of this study, we present four successive models which reflect the modeling process at its different stages with our attempts to make it more realistic by inclusion of more relevant biological phenomena. The small intestine is assumed to be a one-dimensional interval and the bolus moves through its lumen due to migrating myoelectric complex. The bolus is treated as a homogeneous cylinder with a fixed length ℓ and variable radius $R(t)$. The degradation of feedstuffs is the result of volumic and surfacic transformations. This model is based on a system of coupled ordinary differential equations. These equations are solved by a classical numerical integration using Runge-Kutta method. The results of simulation are consistent with the experimental works in the literature (e.g. in the case of purified starch [5]), although more analysis and experimentations are needed to represent the reality more closely.

The second part of this work consists in using the homogenization method to simplify the transport equation and justify the choice of the rate of absorption by intestinal wall [4].

The transport of bolus inside the small intestine is induced by high frequency pulses. These pulses cause rapid variation of the bolus' velocity in the small intestine. We show mathematically that the pulses can be averaged out in an appropriate way therefore the rapidly varying velocity can be replaced by a slowly varying one.

Because of the lack of information about the properties of the small intestine wall, the local absorption rate is not precisely defined. Although, an effective or averaged rate of absorption is determined by help of homogenization methods [6]. To this aim, a 3-D transport-diffusion PDE in the domain Ω_ϵ with a Neumann boundary condition (reflecting the Fourier's law) is defined. The domain Ω_ϵ describes the small intestine. It is a 3-D domain with a small radius r_ϵ and a highly oscillating boundary. The oscillation of its boundary is justified by the presence of the fingerlike villi which cover the inner surface of the small intestine. The unknown of the problem being the the absorbable nutrients, the boundary condition represents the absorption rate by intestinal wall. To justify the choice of a constant absorption rate, our method consists in a passage to the limit from this equation to obtain a 1-D transport equation with a constant averaged rate of absorption .

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