From Lebesgue measure-preserving maps to surface homeomorphisms with pseudo-arc and pseudo-circle attractors

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In 2020 Boyland, de Carvalho and Hall published a seminal paper [6] in which they provided a detailed analysis of boundary dynamics of a parameterized family of sphere homeomorphisms with attractors homeomorphic to the (core) tent inverse limit spaces through the study of prime ends. This result presented the first such study of a chaotic parameterized family on a surface. Prior to this result much attention in Continuum Theory has been directed towards the topological classification of tent inverse limit spaces with the highlight being the result [1] that for any two different parameters in the parameter range $(\sqrt{2}, 2]$ the tent inverse limits (not restricted to its dynamical core) are non-homeomorphic. Motivated by these result we constructed two parameterized families of sphere homeomorphisms varying continuously with the parameter in $[0, 1/2]$ with attractors all homeomorphic to the pseudo-arc and the pseudo-circle respectively, yet presenting rich boundary dynamics [4, 5]. In this talk I will present these constructions that rely on a very useful technique called BBM (Brown-Barge-Martin), which incorporates inverse limits and natural extensions of the underlying bonding maps to embed attractors in manifolds and was presented by Boyland, de Carvalho and Hall in [7]. I will address how this study is intertwined with the study of typical properties of one-dimensional Lebesgue measure-preserving maps on one-dimensional manifolds [2, 3]. This talk is based on joint works with Piotr Oprocha (AGH Krakow & University of Ostrava) as well as Jozef Bobok (CVUT Prague) and Serge Troubetzkoy (Aix Marseille).

References

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