Let $G \curvearrowright X$ be a free action of an amenable group on a compact metrizable space. The shift embeddability problem asks whether there exists a G-equivariant embedding of X into the M-cubical shift $([0,1]^M)^G$, and over the years it has become one of the central questions in the mean dimension theory. The best-known result to date was the theorem of Gutman, Qiao, and Tsukamoto, which provides a satisfying answer for actions of $\mathbb{Z}^d$. In this talk, we will describe how this theorem can be generalized to actions of groups from a larger class. After stating the main results, we will focus on the parts of the proof that required a different approach from that of GQT, specifically:

1) defining the right notion of a tiling,

2) encoding the tilings using a small amount of information, and

3) obtaining the tiling from an a priori weaker property.